Passive Three-Way All-Pass Crossover Networks*

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It is shown that three-way all-pass crossover networks cannot be successfully derived from two-way all-pass networks if easily alignable passive ladder circuits are to be used. New transfer function triples are introduced which do allow the ladder implementation of three-way all-pass networks. Detailed circuits and alignment formulas are given to show how the new crossover networks may be passively realized.

0 INTRODUCTION

Two-way all-pass crossover systems [1] have become an accepted standard in loudspeaker applications because of their flat magnitude response and the fact that they can be passively realized. Unfortunately, as is shown here, if two of these networks are cascaded, the resulting three-way crossover is not all-pass. Even more, there seems to be no way that the known two-way networks can be used to realize three-way all-pass crossovers. The aim of this paper is to present new transfer functions which can be used for this purpose, and to describe circuit topologies and alignment formulas that may be used to implement them.

There are potentially many possible transfer function sets and circuit topologies that could be used to realize three-way all-pass crossovers. The objective here is to find transfer functions which can be realized by economical circuit topologies with efficient alignment algorithms. Nevertheless, higher order networks require lengthy calculations.

1 ALL-PASS NETWORKS

The term all-pass was first applied to crossover networks by Garde [1] and is defined in terms of network transfer ratios. We will use voltage ratios, but the same functions may be viewed as current ratios by using the dual circuit realizations. If $C_1(s), C_2(s), \ldots, C_m(s)$ are the voltage transfer ratios of the individual channels of an m-way crossover network, that network is all-pass if the sum $C_1(s) + C_2(s) + \cdots + C_m(s)$ is an all-pass transfer function in the filter theory sense. This means

$$|C_1(s) + C_2(s) + \cdots + C_m(s)| = 1 \tag{1}$$

for all $s = j\omega$, $\omega \geq 0$. In the usual audio terminology the crossover has a flat summed-channel magnitude response.

Garde [1] derived the sequence of two-way all-pass transfer function pairs defined by

$$C_L(s) = \frac{1}{B_n(s)}, \qquad C_H(s) = \frac{\pm 1}{B_n(1/s)} \tag{2}$$

for n odd and

$$C_L(s) = \frac{1}{B_m^2(s)}, \qquad C_H(s) = \frac{(-1)^m}{B_m^2(s)} \tag{3}$$

for n even, where $B_k(s)$ is the kth-degree Butterworth polynomial, $m = n/2$, and the crossover frequency is 1 rad/s. The first four Butterworth polynomials are listed in Table 1. Networks realizing the odd-order pairs are commonly called Butterworth crossovers and have long been recommended for loudspeaker applications [2] because they exhibit both flat power and flat magnitude response [3]. Networks derived from the even-order functions are also known as Linkwitz–Riley crossovers. Linkwitz [4] advocated them as loudspeaker crossovers because they help maintain a vertically symmetric acoustic radiation pattern. They are also noted [5] for minimizing acoustic response variations caused by the geometric separation and phase difference between the drivers in a two-way loudspeaker system.

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The popularity of the all-pass networks is clinched by the fact that they can be realized by simple passive circuitry, that is, resistance-terminated LC ladders. The circuits and alignments for the Butterworth networks can be found in any filter design handbook, and [6] contains the same information for the Linkwitz–Riley networks.

2 THREE-WAY NETWORKS

The low-pass, bandpass, and high-pass transfer functions of a three-way crossover are denoted by $C_L(s)$, $C_B(s)$, and $C_H(s)$, respectively. If $\omega_1 < \omega_2$ are the radian crossover frequencies, then it is assumed that the transfer functions are frequency normalized to the geometric mean ω_0 of ω_1 and ω_2:

$$\omega_0 = \sqrt{\omega_1 \omega_2} \ . \tag{4}$$

The normalized radian crossover frequencies r and R are then defined by

$$r = \omega_1/\omega_0 \tag{5}$$

$$R = \omega_2/\omega_0 \ . \tag{6}$$

It follows that

$$Rr = 1 \ . \tag{7}$$

In spite of Eq. (7), we will continue to use both r and R in formulas so that the role of the two crossover frequencies is clear at a glance.

The behavior of a three-way crossover depends strongly on the separation between the two crossover frequencies. Because of this we define the crossover frequency spread as ω_2/ω_1, which with Eqs. (5)–(7) becomes

$$\text{spread} = R^2 \ . \tag{8}$$

Note that $R = 2$ corresponds to a two-octave spread and $R = 3$ to approximately three octaves.

We are interested in deriving transfer functions for three-way all-pass crossovers which can be realized with economical passive circuitry and efficient alignment procedures. The most direct approach would be to somehow use the two-way all-pass functions. We now show that this is not possible by any straightforward technique.

Table 1. Butterworth polynomials through degree 4.

$B_1(x)$	$= 1 + x$
$B_2(x)$	$= 1 + \sqrt{2}\, x + x^2$
$B_3(x)$	$= 1 + 2x + 2x^2 + x^3$
$B_4(x)$	$= 1 + \sqrt{4 + 2\sqrt{2}}\, x + (2 + \sqrt{2})x^2 + \sqrt{4 + 2\sqrt{2}}\, x^3 + x^4$

3 CASCADED TWO-WAY NETWORKS

The simplest way to obtain a three-way network is to cascade two-way networks. The cascade is formed by using the output of the first-stage high-pass section as the input to the second stage. Now if both stages are two-way all-pass networks, is this an all-pass crossover? If loading effects are ignored, the relevant transfer functions are easily found starting with Eq. (2) or Eq. (3). For example, with $n = 2$,

$$C_L(s) = \frac{1}{B_1^2(s/r)} \tag{9}$$

$$C_B(s) = \frac{-1}{B_1^2(r/s) B_1^2(s/R)} \tag{10}$$

$$C_H(s) = \frac{1}{B_1^2(r/s) B_1^2(R/s)} \tag{11}$$

while if $n = 3$

$$C_L(s) = \frac{1}{B_3(s/r)} \tag{12}$$

$$C_B(s) = \frac{\pm 1}{B_3(r/s) B_3(s/R)} \tag{13}$$

$$C_H(s) = \frac{1}{B_3(r/s) B_3(R/s)} \tag{14}$$

These functions can be checked for all-pass behavior by graphing the decibel magnitude

$$M(\omega) = 20 \log |C_L(j\omega) + C_B(j\omega) + C_H(j\omega)|$$

versus the frequency ω. If they sum to all-pass, then the graph should be the 0-dB line. The results for the second- and third-order functions are shown in Fig. 1.

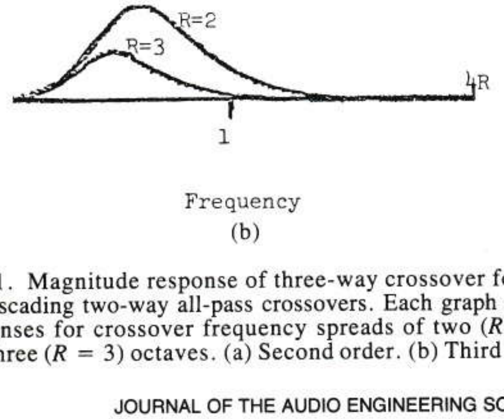


Fig. 1. Magnitude response of three-way crossover formed by cascading two-way all-pass crossovers. Each graph shows responses for crossover frequency spreads of two ($R = 2$) and three ($R = 3$) octaves. (a) Second order. (b) Third order.

Each graph contains two responses, one using a two-octave crossover frequency spread ($R = 2$) and the other three octaves ($R = 3$). The frequency scale is logarithmic and runs from two octaves below the low crossover frequency to two octaves above the high one. The vertical axis covers the range from −3 dB to 3 dB. When more than one polarity choice is possible, the one giving the least response variation is used.

These figures clearly show that cascading does not yield an all-pass crossover. The results for higher orders are similar. Linkwitz–Riley (even-order all-pass) crossovers have a fairly flat magnitude response for large crossover frequency spreads, as the $R = 3$ graph in Fig. 1(a) shows. This is somewhat deceptive if each stage is realized by passive LC ladders, because loading effects produce even larger variation. Thus cascading gives at best only an approximation to all-pass response, and other approaches must be considered if exact all-pass networks are desired.

The reader will note that the above claims are not true if the three-way network is obtained by cascading two first-order networks. In this case the three-way network is not only all-pass, but satisfies the much stronger constant-voltage property introduced in Small [7]. The shortcomings of first-order two-way networks are widely known [8, sec. 1], and they apply equally well when more than two channels are involved. Thus our main effort here is directed toward finding "higher order" crossovers.

4 PARALLEL FILTERS

Another approach is to use the two-way all-pass functions to realize separately low-pass, bandpass, and high-pass functions, and to realize the crossover with these three filters in parallel. To achieve such a design most efficiently, the low-pass two-way function is denormalized to r to obtain $C_L(s)$ and the two-way high-pass is denormalized to R to get $C_H(s)$. For example, based on the second-order functions,

$$C_L(s) = \frac{1}{B_1^2(s/r)} \tag{15-1}$$

$$C_H(s) = \frac{1}{B_1^2(R/s)} \tag{15-3}$$

and for the third-order functions,

$$C_L(s) = \frac{1}{B_3(s/r)} \tag{16-1}$$

$$C_H(s) = \frac{1}{B_3(R/s)} . \tag{16-3}$$

There are two simple ways to get a bandpass function. One is to use the low-pass to bandpass frequency transformation

$$s \rightarrow (s^2 + \omega_B^2)/(Bs)$$

on the two-way low-pass function, where ω_B is the desired center frequency and B the bandwidth of the resulting bandpass function. We choose $\omega_B = \omega_0 = 1$ and $B = R - r$ as the normalized values for these constants. For the second- and third-order networks this gives

$$C_B(s) = \frac{\pm 1}{B_1^2[(s^2 + 1)/(Bs)]} \tag{15-2T}$$

and

$$C_B(s) = \frac{\pm 1}{B_3[(s^2 + 1)/(Bs)]} \tag{16-2T}$$

respectively. The advantage of this technique is that the alignment for the bandpass circuit is completely determined by the low-pass one, that is, it is computationally efficient. However, as Fig. 2 shows, the resulting three-way networks are far from all-pass. The graphing specifications here are the same as before. The variation is similar if the higher order functions are used, and changing values of ω_B and B does not significantly improve it.

The second way to get the bandpass section is to cascade low- and high-pass filters. This method is less efficient computationally in passive realizations because the bandpass must be separately aligned in order to account for loading effects. The second- and third-order bandpass functions are then

$$C_B(s) = \frac{-1}{B_1^2(r/s)B_1^2(s/R)} \tag{15-2C}$$

and

$$C_B(s) = \frac{\pm 1}{B_3(r/s)B_3(s/R)} \tag{16-2C}$$

respectively. Fig. 3 shows that the use of this bandpass function still does not result in all-pass response. Similar variations occur using the higher order two-way networks.

5 IMPLICATIONS

Other practical approaches to coaxing all-pass behavior from two-way-based networks come to nothing. For example, independently varying the gains of the three channels does not flatten the response, it merely changes the ripple pattern. From these considerations it may be concluded that three-way all-pass crossovers are to be most efficiently obtained by deriving new transfer functions with the all-pass condition (1) as the governing constraint.

The objective then is to derive triples of functions satisfying Eq. (1), which are realizable by economical passive circuitry and efficient alignment algorithms. The easiest way to attain the latter conditions is to assume that the complete crossover can be derived from a single low-pass filter function by the use of cascading and frequency transformations. To realize and align the low-pass function efficiently, it is desirable to assume that it is an all-pole function. This will also maximize the stopband attenuation of the resulting crossover.

6 TRANSFER FUNCTION FORM

We will assume

$$C_L(s) = \frac{1}{P(s/r)} \tag{17}$$

where

$$P(s) = s^n + a_1 s^{n-1} + \cdots + a_{n-1}s + 1 \tag{18}$$

is a Hurwitz polynomial. Then the low-pass section can be realized by an elementary ladder. The full transfer function complement can then be obtained in various ways, but we will concentrate on the three methods used in Sections 3 and 4.

1) Cascade derived:

$$C_L(s) = \frac{1}{P(s/r)} \tag{19-L}$$

$$C_B(s) = \frac{\pm 1}{P(r/s)P(s/R)} \tag{19-B}$$

$$C_H(s) = \frac{1}{P(r/s)P(R/s)} \; . \tag{19-H}$$

2) Transformation bandpass and high-pass:

$$C_L(s) = \frac{1}{P(s/r)} \tag{20-L}$$

$$C_B(s) = \frac{\pm 1}{P[(s^2 + \omega_B^2)/Bs]} \tag{20-B}$$

$$C_H(s) = \frac{1}{P(R/s)} \tag{20-H}$$

with $\omega_B = \omega_0 (= 1)$ and $B = R - r$.

3) Cascaded bandpass, transformation high-pass:

$$C_L(s) = \frac{1}{P(s/r)} \tag{21-L}$$

$$C_B(s) = \frac{\pm 1}{P(r/s)P(s/R)} \tag{21-B}$$

$$C_H(s) = \frac{1}{P(R/s)} \; . \tag{21-H}$$

Option 2) has an apparent efficiency advantage because both the bandpass and the high-pass alignments are automatically determined by the low-pass alignment.

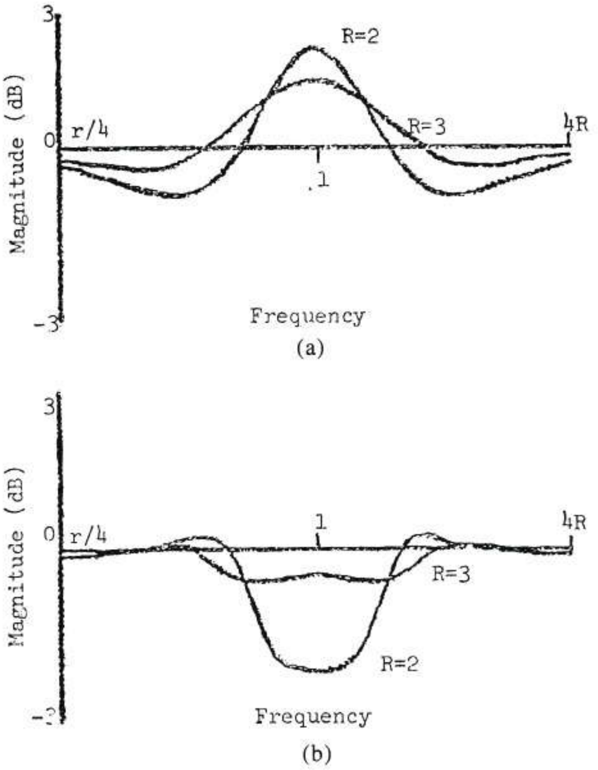


Fig. 2. Magnitude response of three-way crossover formed by parallel connection of two-way all-pass crossover filters. Here the bandpass is obtained by frequency transformation. Each graph shows responses for crossover frequency spreads of two ($R = 2$) and three ($R = 3$) octaves. (a) Second order. (b) Third order.

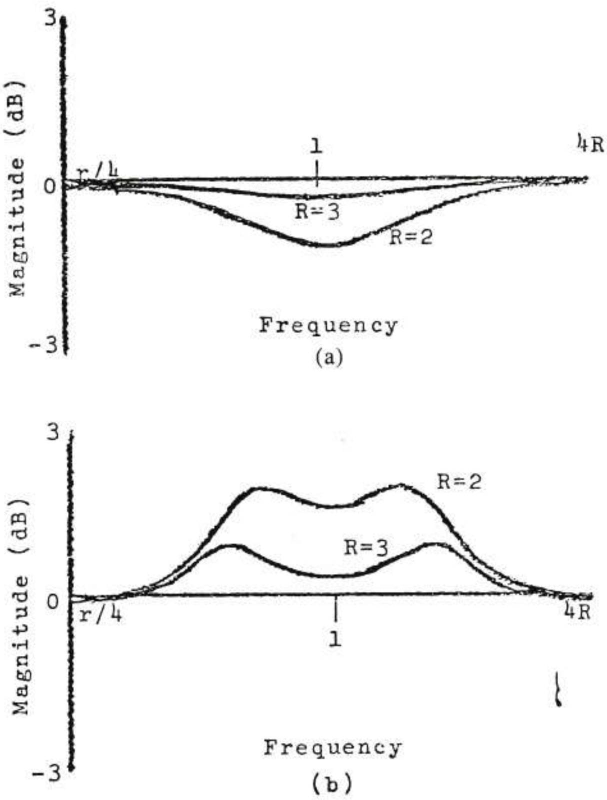


Fig. 3. Magnitude response of three-way crossover formed by parallel connection of two-way all-pass crossover filters. Here the bandpass is obtained by cascading. Each graph shows responses for crossover frequency spreads of two ($R = 2$) and three ($R = 3$) octaves. (a) Second order. (b) Third order.

Option 1) has the advantage of a better stopband attenuation in the high-pass section.

To derive an all-pass transfer function triple based on these options we must find values for $a_1, a_2, \ldots, a_{n-1}$ so that the equation

$$|C_L(j\omega) + C_B(j\omega) + C_H(j\omega)| = 1 \tag{22}$$

is satisfied for all $\omega \geqslant$. If $C_L(j\omega) + C_B(j\omega) + C_H(j\omega)$ is expressed as a single rational function and its squared magnitude is taken, Eq. (22) reduces to the statement that two polynomials in ω must be identical. This gives rise to $4n - 1$ equations in $a_1, a_2, \ldots, a_{n-1}, r, R$, which must be solved for the a_k with r and R as fixed parameters. Because of symmetry, options 2) and 3) yield only $2n$ independent equations, but even so the number of equations is still too great to expect a solution.

The number of equations arising from options 1) and 3) can be substantially reduced by replacing P in the bandpass functions with

$$P_0(s) = s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + 1 . \tag{23}$$

Note that P_0 is obtained from P by reversing the coefficient order. This additional functional symmetry reduces the number of equations to $2n - 1$ for option 1) and to n for 3). Thus if one more variable can be introduced in the modified option 3), that form will give a system of n equations in n unknowns and good prospects for a solution.

A successful nth variable turns out to be a bandpass gain factor h, that is, we assume the functional forms

$$C_L(s) = \frac{1}{P(s/r)} \tag{24}$$

$$C_B(s) = \frac{h}{P_0(r/s)P_0(s/R)} \tag{25}$$

$$C_H(s) = \frac{1}{P(R/s)} . \tag{26}$$

The problem now is to find $a_1, a_2, \ldots, a_{n-1}, h$ so that Eqs. (24)–(26) satisfy Eq. (22), and so that $P(s)$ is a Hurwitz polynomial. The parameter h can be either positive or negative to indicate the polarity to be observed in the bandpass section. A crossover with these transfer functions can be assigned an order, namely n, just as for the standard two-way crossovers. Further, the order has the same significance in that the stopband asymptotic attenuation rate will be $6n$ dB per octave in all channels for the nth-order network.

7 ALTERNATIVE FORMS

Before proceeding, it should be emphasized that the transfer function forms chosen here are not the only ones that can be successful. As a matter of fact, if the cascade-derived option 1) is modified to

$$C_L(s) = \frac{1}{P(s/r)}$$

$$C_B(s) = \frac{h}{P_0(r/s)Q(s/R)}$$

$$C_H(s) = \frac{1}{P_0(r/s)Q_0(R/s)}$$

where Q is a second Hurwitz polynomial of degree n, then usable solutions exist, at least for $n = 2$. However, the solution is more difficult to find for a given n because $2n - 1$ rather than n equations must be solved.

8 THREE-WAY ALL-PASS FUNCTIONS

The system of equations resulting from Eqs. (22)–(26) is nonlinear and contains the additional variable R. Thus the solutions will depend on the crossover frequency spread. In other words, we should not expect to be able to use the same polynomial $P(s)$ for all three-way all-pass crossovers of a given order. The form of the system of equations which must be solved depends on the order n, and it does not appear possible to solve it for arbitrary n. We will describe solutions through order 4, which should cover most conceivable audio applications.

8.1 First Order

This case is included simply for completeness. Because

$$P(s) = s + 1 \tag{27}$$

is the only polynomial of degree 1 and the form of Eq. (23), h is the only free parameter in the system of equations. Substituting Eqs. (23)–(25) into Eq. (22) gives

$$|s^2 + (2r + hR)s + 1| = |s^2 + (R + r)s + 1| .$$

The polynomials inside the absolute values are the same if

$$2r + hR = R + r,$$

which yields

$$h = 1 - r^2 . \tag{28}$$

With this choice of h and P, the crossover defined by Eqs. (24)–(26) has both constant amplitude and zero phase, that is, it defines a three-channel constant-voltage crossover.

As mentioned in Section 3, a first-order three-way constant-voltage crossover can also be realized by cascading two-way sections. It is preferable to the one described here for two reasons. First it can be passively implemented with less computation, because a gain factor is unnecessary and no bandpass filter is required. There are no loading effects between the two stages because a first-order two-way network is a constant-resistance network. This assures that its combined power response is flat, which is the second reason it is to be preferred.

8.2 Second Order

The required polynomial form is

$$P(s) = s^2 + as + 1 . \quad (29)$$

In this case P and P_0 are identical and a and h are the unknowns. Substitution of Eqs. (23)–(25) into Eq. (22) yields

$$|s^4 + ars^3 + (2r^2 + hR^2)s^2 + ars + 1|$$
$$= |s^4 + a(R + r)s^3 + (R^2 + a^2 + r^2)s^2 + a(R + r)s + 1| . \quad (30)$$

This equality cannot be satisfied by making the two s polynomials equal, so the crossover cannot have the constant-voltage property. To obtain equations in a and h, Eq. (30) must be converted to a magnitude-squared expression using $s = \omega j$. This gives

$$[1 - (2r^2 + hR^2)\omega^2 + \omega^4]^2 + a^2r^2(\omega - \omega^3)^2$$
$$= [1 - (R^2 + a^2 + r^2)\omega^2 + \omega^4]^2 + a^2(R + r)^2(\omega - \omega^3)^2 \quad (31)$$

which must hold for all $\omega \geqslant 0$. Hence, at $\omega = 1$,

$$(2 + 2r^2 + hR^2)^2 = (2 + R^2 + a^2 + r^2)^2 .$$

This implies either

$$2 + 2r^2 + hR^2 = 2 + R^2 + a^2 + r^2 \quad (32)$$

or

$$2 + 2r^2 + hR^2 = - 2 - R^2 - a^2 - r^2 . \quad (33)$$

Next the ω^2 coefficients of the polynomials in Eq. (31) must be equal, so

$$a^2r^2 - 2(2r^2 + hR^2)$$
$$= a^2(R + r)^2 - 2(R^2 + a^2 + r^2) . \quad (34)$$

Now condition (32) is incompatible with this as it leads to $R = 0$. Thus Eq. (33) is the only choice leading to a solution. Solving Eqs. (33) and (34) simultaneously gives

$$a = \frac{2(R^2 - 1)}{R\sqrt{R^2 - 2}} \quad (35)$$

and

$$h = - [1 + (a^2 - 4)r^2 + 3r^4] . \quad (36)$$

From Eq. (35) we see that there is a second-order all-pass crossover for all values of $R > \sqrt{2}$. This means that the crossover spread must be more than an octave. This is not to say that an all-pass crossover does not exist for $R \leqslant \sqrt{2}$, but if one does, its transfer functions do not have the form of Eqs. (23)–(26).

It is interesting to note that $a \rightarrow 2$ as $R \rightarrow \infty$. This can be interpreted as saying that the transfer functions found here approach the known two-way all-pass transfer functions. In this sense the new three-channel crossover is a natural analog of the two-way all-pass crossover.

Formula (35) implies that $a > 2$ for all allowable R. This fact and Eq. (36) show that the multiplier h is always negative, that is, the bandpass polarity must be reversed. This is consistent with the fact that the two-way second-order all-pass network uses opposite polarities in adjacent channels.

8.3 Third Order

For third-order crossovers the polynomial P has the form

$$P(s) = s^3 + as^2 + bs + 1 . \quad (37)$$

This is the first instance in which P and P_0 may be distinct. This fact complicates the derivation, so the details are relegated to the Appendix, where it is shown that there are two possible solutions.

The values of a, b, and h for the first can be obtained by finding b as a positive root of

$$b^5 + (R^4 + 4r^2)b^3 - 8b^2 + 4R^2b - 8(R^4 + r^2) = 0 \quad (38)$$

and then computing a and h from

$$a = b^2/2 \quad (39)$$

and

$$h = 1 + (a^2 - 2b)r^2 + 2ar^4 - r^6 . \quad (40)$$

Table 2 lists solutions for R values from 1.4 to 5 in 0.1 steps for low R and 0.2 steps for larger ones.

The second solution results from finding b as a positive root of

$$b^5 - (R^4 + 4r^2)b^3 - 8b^2 + (4R^2 + 8r^4)b + 8(R^4 + r^2) = 0 \quad (41)$$

and calculating a and h from

$$a = b^2/2 \tag{42}$$

and

$$h = -1 - (a^2 - 2b)r^2 + 2ar^4 - 3r^6 . \tag{43}$$

Solutions are listed in Table 3 for R between 1.7 and 5 in the same format as Table 2.

Solutions of the first system actually exist for any $R > 1$, but were not tabulated for the smaller R values because it is expected that they would rarely be used in practice. Solutions of the second system do not exist for $R \leq 1.7$. (This value is approximate; the exact value is between 1.6 and 1.7.) It is clear from the tabulated data that the bandpass polarity is observed in the first solution and reversed in the second. This is consistent with the situation for two-way all-pass crossovers, that is, all-pass response can be obtained for third-order crossovers whether the polarity of adjacent channels is the same or opposite. For either crossover it is true that $a \to 2$ and $b \to 2$ as $R \to \infty$. Thus these transfer functions are natural three-channel extensions of the two-way all-pass functions.

8.4 Fourth Order

The fourth-order transfer functions require a fourth-degree polynomial of the form

$$P(s) = s^4 + as^3 + bs^2 + cs + 1 . \tag{44}$$

Table 2. Parameters for positive-polarity bandpass third-order three-way all-pass crossover.

R	a	b	h
1.4	1.4794	1.7201	0.9988
1.5	1.5261	1.7470	0.9973
1.6	1.5683	1.7711	0.9962
1.7	1.6064	1.7924	0.9956
1.8	1.6405	1.8113	0.9956
1.9	1.6710	1.8381	0.9958
2.0	1.6983	1.8430	0.9963
2.1	1.7228	1.8562	0.9968
2.2	1.7446	1.8679	0.9971
2.3	1.7642	1.8784	0.9975
2.4	1.7817	1.8877	0.9979
2.5	1.7975	1.8961	0.9982
2.6	1.8118	1.9036	0.9985
2.7	1.8246	1.9103	0.9987
2.8	1.8363	1.9164	0.9989
2.9	1.8468	1.9219	0.9990
3.0	1.8564	1.9269	0.9992
3.2	1.8732	1.9355	0.9994
3.4	1.8872	1.9428	0.9996
3.6	1.8991	1.9489	0.9997
3.8	1.9092	1.9541	0.9998
4.0	1.9179	1.9585	0.9998
4.2	1.9254	1.9623	0.9999
4.4	1.9319	1.9657	0.9999
4.6	1.9370	1.9686	0.9999
4.8	1.9427	1.9711	0.9999
5.0	1.9471	1.9733	0.9999

Again, the derivation is lengthy and so is carried out in the Appendix. The result is that there are two possible solutions. The relevant values of a, b, c, and h for the first can be found from the equations

$$c^2 - 2b = 0 \tag{45}$$

$$(b^2 - 2ac + 2)R^8 - 2R^8h - 2 = 0 \tag{46}$$

$$(a^2 - 2b)R^{10} + (a^2 - b)(b^2 - 2ac + 2)R^6 + 2bR^2 - 2ac = 0 \tag{47}$$

$$(b^2 - 2ac)R^6 - 2(a^2 - 2b)R^4 - 2(b^2 - 2ac)R^2 - 4b = 0 . \tag{48}$$

Solutions of this system exist for any $R > 1.2$ (approximately) and are tabulated in Table 4 for R between 1.4 and 5. As can be seen from the table, $a \to 2\sqrt{2}$, $b \to 4$, $c \to 2\sqrt{2}$, and $h \to 1$ as $R \to \infty$, which means that this crossover is a natural extension of the fourth-order two-way all-pass crossover.

A second solution can be found by solving Eqs. (45)–(47) and

$$(b^2 - 2ac + 4)R^8 + 2(a^2 - 2b)R^6 + 2(b^2 - 2ac + 4)R^4 - 4bR^2 + 4 = 0 . \tag{49}$$

On an intuitive basis we should not expect this second solution, because there is no corresponding two-way analog. If this solution is examined as $R \to \infty$, it is found that $a, b, c \to 2$. In other words, the analogous two-way transfer functions would have the denominator $(s^4 + 2s^3 + 2s^2 + 2s + 1) = (s + 1)^2 (s^2 + 1)$, which does not yield a passively realizable transfer function [9, p. 343]. This means that the three-way functions can be expected to have a high-Q second-order section, which complicates the problem of circuit design. Further, there is an approximately 180° phase difference between adjacent channels, a feature that does not bode well for its use as a loudspeaker crossover. For these reasons it was decided not to tabulate its parameters. The discovery of this unexpected all-pass crossover contrasts sharply with the situation for two-channel crossovers, where the passive all-pass constraint yields a unique crossover for each even order [10].

8.5 Higher Order All-Pass Crossovers

The techniques implicit in the above derivations can be used to find transfer functions of higher order all-pass crossovers, but the relevant equations appear to increase dramatically in their complexity with the order. The easier procedure for higher orders is to use cascaded two-way all-pass networks with a crossover frequency spread of at least three octaves. This does not yield exact all-pass response, but is within a fraction of a decibel and eliminates the need for very lengthy calculations. Even so, the even-order cases will involve additional calculations because alignments must be adjusted to account for loading effects.

Table 3. Parameters for negative-polarity bandpass third-order three-way all-pass crossovers.

R	a	b	h
1.7	2.7708	2.3541	−1.4882
1.8	2.5580	2.2618	−1.2242
1.9	2.4576	2.2170	−1.1314
2.0	2.3919	2.1872	−1.0846
2.1	2.3435	2.1649	−1.0575
2.2	2.3055	2.1473	−1.0405
2.3	2.2747	2.1329	−1.0294
2.4	2.2489	2.1208	−1.0218
2.5	2.2270	2.1105	−1.0650
2.6	2.2082	2.1015	−1.0126
2.7	2.1918	2.0937	−1.0098
2.8	2.1773	2.0868	−1.0077
2.9	2.1646	2.0807	−1.0061
3.0	2.1532	2.0752	−1.0049
3.2	2.1338	2.0658	−1.0033
3.4	2.1180	2.0582	−1.0022
3.6	2.1049	2.0518	−1.0016
3.8	2.0939	2.0464	−1.0011
4.0	2.0846	2.0418	−1.0008
4.2	2.0766	2.0379	−1.0006
4.4	2.0697	2.0345	−1.0004
4.6	2.0640	2.0310	−1.0003
4.8	2.0584	2.0290	−1.0003
5.0	2.0538	2.0267	−1.0002

Table 4. Parameters for fourth-order three-way all-pass crossovers.

R	a	b	c	h
1.4	2.4304	3.7515	2.7392	1.3120
1.5	2.5051	3.7544	2.7402	1.1444
1.6	2.5651	3.7816	2.7501	1.0726
1.7	2.6130	3.8126	2.7614	1.0382
1.8	2.6512	3.8415	2.7718	1.0247
1.9	2.6819	3.8666	2.7809	1.0114
2.0	2.7066	3.8879	2.7885	1.0064
2.1	2.7766	3.9056	2.7948	1.0037
2.2	2.7428	3.9202	2.8001	1.0021
2.3	2.7561	3.9323	2.8044	1.0012
2.4	2.7669	3.9423	2.8080	1.0007
2.5	2.7758	3.9506	2.8109	1.0004
2.6	2.7832	3.9575	2.8134	1.0002
2.7	2.7894	3.9633	2.8154	1.0001
2.8	2.7946	3.9681	2.8171	1.0001
2.9	2.7989	3.9722	2.8186	1.0000
3.0	2.8026	3.9757	2.8198	1.0000
3.2	2.8084	3.9811	2.8217	1.0000
3.4	2.8127	3.9852	2.8232	1.0000
3.6	2.8159	3.9882	2.8242	1.0000
3.8	2.8183	3.9905	2.8251	1.0000
4.0	2.8202	3.9922	2.8257	1.0000
4.2	2.8216	3.9936	2.8262	1.0000
4.4	2.8228	3.9947	2.8265	1.0000
4.6	2.8237	3.9955	2.8269	1.0000
4.8	2.8244	3.9962	2.8271	1.0000
5.0	2.8250	3.9968	2.8273	1.0000

9 POWER RESPONSE

A crossover has the constant-power property if its channel transfer functions $C_1(s), \ldots, C_m(s)$ satisfy

$$|C_1(j\omega)|^2 + |C_2(j\omega)|^2 + \cdots + |C_m(j\omega)|^2 = 1 \tag{50}$$

for all $\omega \geq 0$. This property is desirable in loudspeaker crossovers because such a network will not add distortion to the overall acoustic power response. The odd-order two-way all-pass networks have this property, but none of the new three-way networks does. Fig. 4 shows the power responses of the new second-, third-, and fourth-order networks, respectively. The same graph

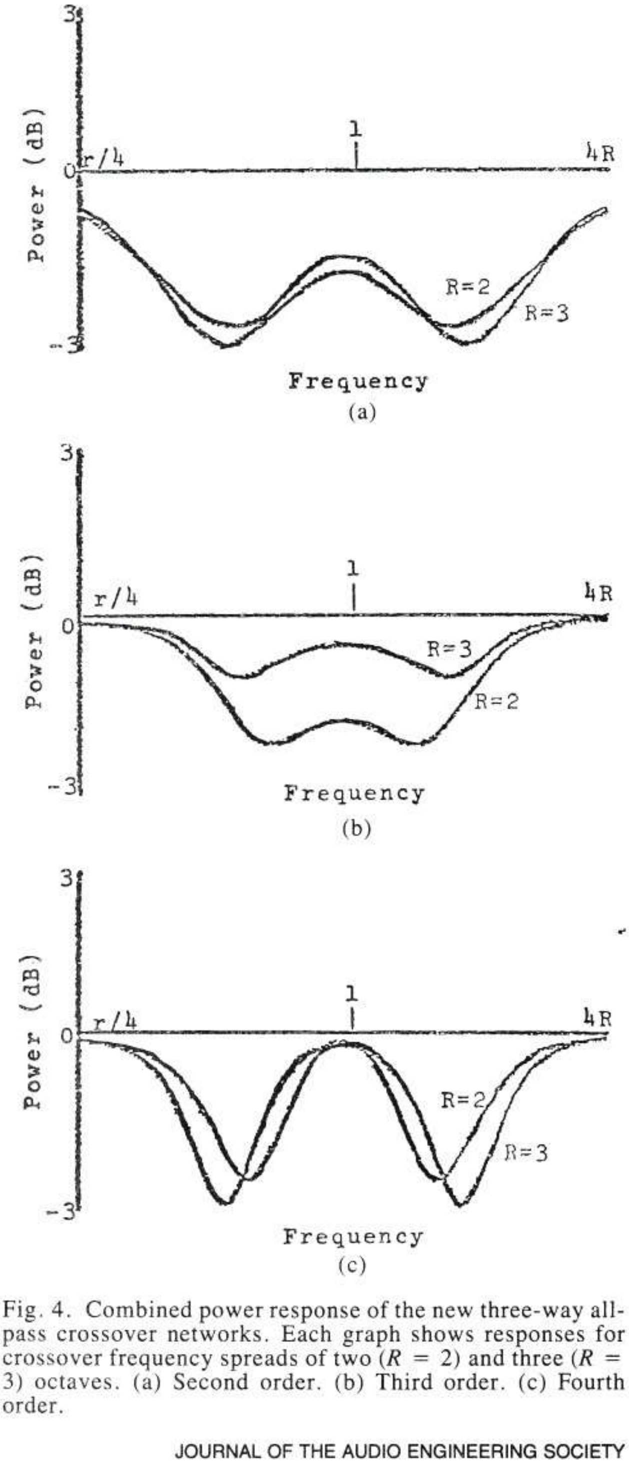


Fig. 4. Combined power response of the new three-way all-pass crossover networks. Each graph shows responses for crossover frequency spreads of two (R = 2) and three (R = 3) octaves. (a) Second order. (b) Third order. (c) Fourth order.

specifications are used as in earlier figures.

It appears that the only hope of realizing three-way networks with both the constant-power and all-pass properties is to abandon ladder circuits. Thus it is likely that the most practical approach would be to permit active realizations.

10 CROSSOVER FREQUENCY SPREAD

As already noted, the three-way all-pass transfer functions (24)–(26) depend on the crossover frequency spread. In general for small values of R ($R < 3$), the defining polynomial $P(s)$ is quite different from the corresponding two-way all-pass polynomial. This is why the attempts to obtain all-pass response from the known two-way functions failed. However, for large crossover frequency spreads ($R \geqslant 3$), $P(s)$ is quite close to its two-way counterpart. In practice this means that the two-way polynomial could be used in place of $P(s)$ without wide response variations resulting. In particular, it appears that the ripple caused by such an approximation when $R \geqslant 3$ is smaller than 1 dB. Because polynomial coefficients do not have to be calculated, the approximation is computationally more efficient. However, P must be found for small spreads, and the procedure is no more difficult with large ones, so we do not make use of this approximation.

11 PASSIVE REALIZATION

The various crossover transfer functions introduced above can be realized passively using standard circuit topologies, but there are several complications. First, many of the required alignments are new and so are not tabulated in any handbook. Second, realization of the cascaded bandpass filter is not explicitly discussed in circuit design references. Third, provisions for realizing the gain factor h must be incorporated. Therefore a self-contained treatment of the passive realization of the new networks is included here. This verifies the claim that the new crossovers are indeed passively realizable.

The resistance-terminated LC ladder circuit is used for passive realization. This topology is traditional in crossover design, and alignment formulas can be straightforwardly derived; but two facts regarding its use must be kept in mind. First, it should ideally be driven by a zero resistance source and second, its termination should be resistive. It will deviate from its theoretical response in proportion to variations from these states. In loudspeaker applications, the first requirement can be satisfied practically by the use of a high-damping-factor amplifier.

The second requirement is more difficult to satisfy, because a typical loudspeaker does not present a resistive impedance and cannot always be equalized to do so. The practical goal should be to equalize to the flattest impedance practicable over the widest possible frequency band containing the crossover frequencies. Theoretically such equalization can be achieved exactly for any loudspeaker fitting the Thiele–Small models by paralleling the driver terminals with an LCR circuit whose impedance is the dual of the loudspeaker's with respect to the driver's dc resistance. However, the results are practically restricted by the accuracy of the model, as well as by nonideal component behavior and component size limitations in the equalizing LCR network.

Crossovers can be formed by either parallel or series connection of filters. We regard the transfer functions as voltage ratios and hence use parallel connections. The series connection can be obtained by regarding the transfer functions as current ratios and using the duals of the circuits presented. All the circuits described here have a natural positive polarity. If the crossover of interest calls for a negative bandpass polarity, reverse the natural polarity of its termination.

11.1 Low-Pass Circuit

The low-pass crossover section is realized using the circuits and alignment formulas in Fig. 5 for the first through the fourth order, respectively. These circuits, as well as all subsequent ones, have component values which are frequency normalized to 1 rad/s and impedance normalized to a 1-Ω terminating resistance. The resulting filter realizes the $C_L(s)$ in Eq. (24). The circuits may be denormalized to a low-section load resistance of R_W and a lower radian crossover frequency ω_1 by multiplying each inductor value by R_W/ω_1 and dividing each capacitor value by $R_W\omega_1$.

11.2 High-Pass Circuit

The frequency- and impedance-normalized high-pass circuit and alignment formulas are obtained by applying the low-pass to high-pass frequency transformation $s \rightarrow 1/s$ to the circuits in the last section. The resulting circuit is found by replacing each inductor by a capacitor and each capacitor by an inductor. The normalized alignment formulas are the reciprocals of the low-pass formulas with inductor and capacitor names interchanged to fit the circuit. Fig. 6 gives the results for orders 1 through 4, respectively. This high-pass filter may then be denormalized to the high section terminating resistance R_T and the high radian crossover frequency ω_2, just as was done in the low-pass case, that is, by multiplying each inductor value by R_T/ω_2 and dividing each capacitor value by $R_T\omega_2$.

11.3 Bandpass Circuit

Figs. 7–10 show the bandpass circuit topologies and their alignment formulas for orders 1 through 4, respectively. These circuits are denormalized by multiplying each inductor value by R_B/ω_0, dividing each capacitor value by $R_B\omega_0$, and multiplying R_{an} by R_B, where R_B is the desired terminating resistance and ω_0 is given by Eq. (4).

The correct bandpass gain factor h is set in each of the above circuits by the value of the resistor R_{an}. An alternative approach is worth considering when the crossover is part of a loudspeaker system. Find the bandpass with $R_{an} = 0$ in both the design formulas and the circuit. It then has an excess gain E in the amount

$$E = 20 \log (K/H) \quad \text{dB}$$

where H is defined in Table 5 for each order, and K is found among the design formulas for the circuit used. If the midrange driver nominal sensitivity is S_M, then the value of h may be automatically accounted for by using $S_M + E$ as its effective value in the sensitivity matching phase of design. E is always positive, so this alternative allows the possibility of using a midrange driver of lower nominal sensitivity rather than that of the woofer and tweeter.

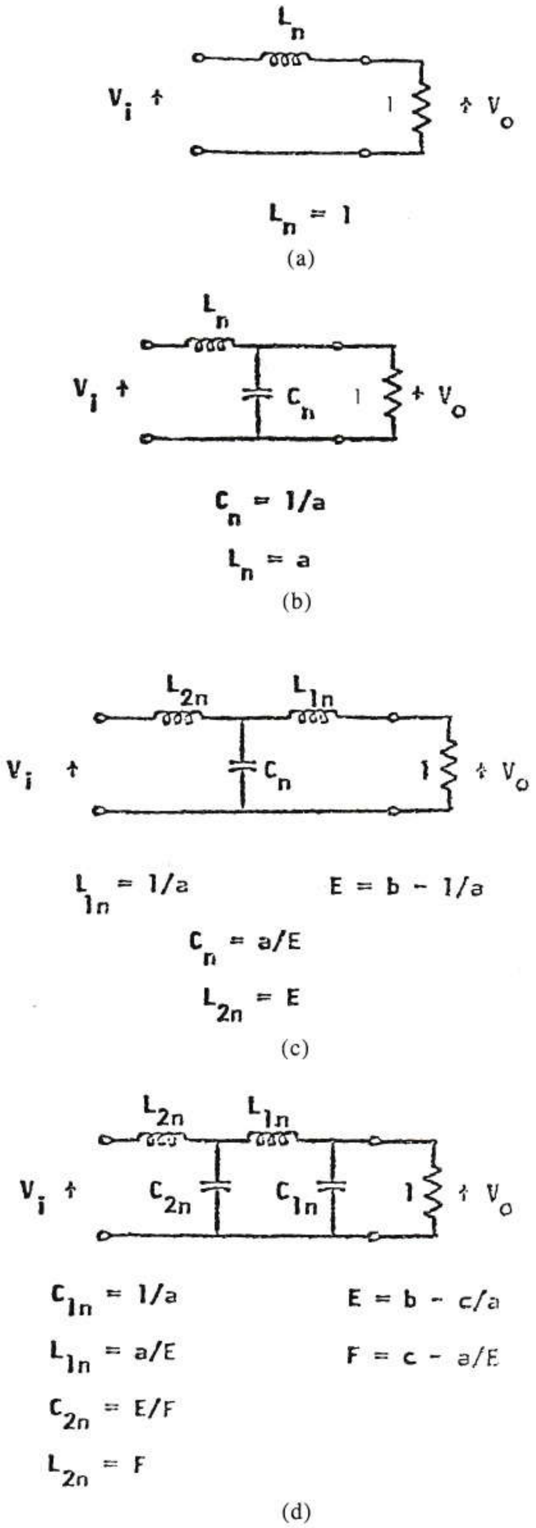


Fig. 5. Low-pass filter circuits and alignment formulas for the new crossovers. (a) First order. (b) Second order, parameter a from Eq. (35). (c) Third order, parameters a, b from Table 2 or 3. (d) Fourth order, parameters a, b, c from Table 4.

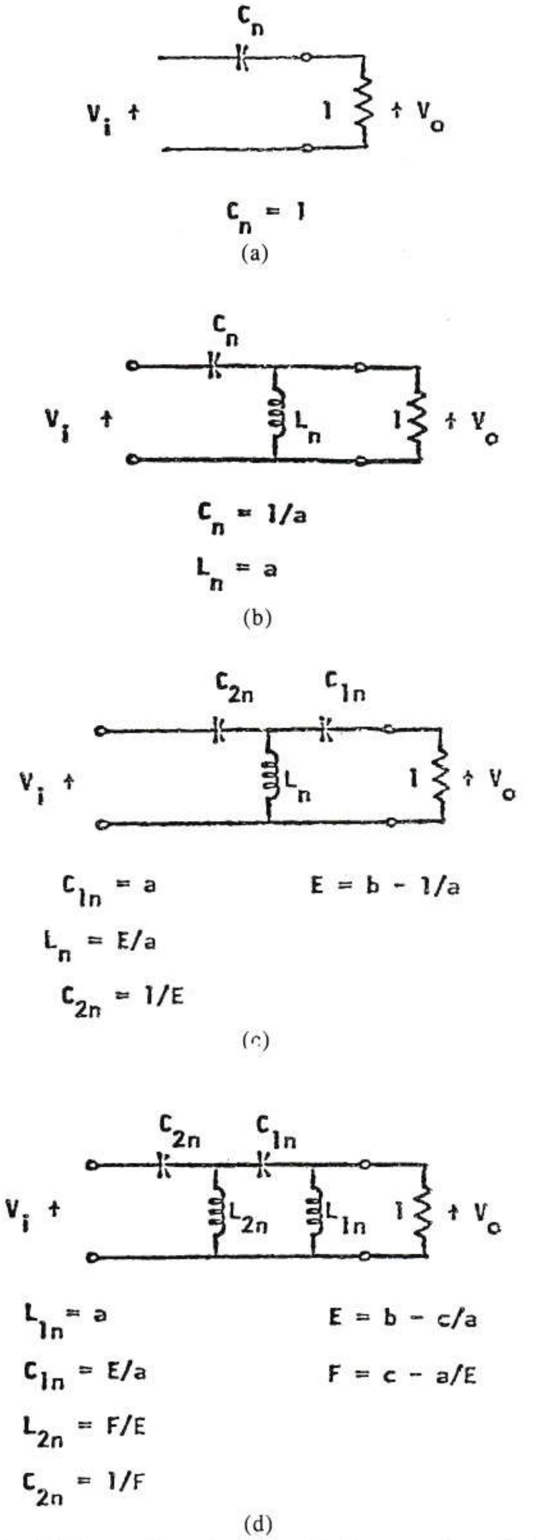
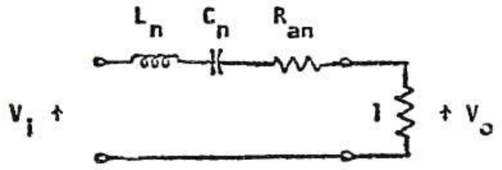


Fig. 6. High-pass filter circuits and alignment formulas for the new crossovers. (a) First order. (b) Second order. (c) Third order. (d) Fourth order. Relevant a, b, c parameters chosen as in Fig. 5.

12 DESIGN PROCEDURE

The step-by-step design procedure for the new networks is as follows. Choose a crossover order (1, 2, 3, 4) and crossover frequencies $f_1 < f_2$. From $R = \sqrt{f_2/f_1}$ and $r = 1/R$ determine the polynomial coefficients and gain factor using Eq. (28) for order 1, Eqs. (35) and (36) for order 2, Table 2 or 3 for order 3, or Table 4 for order 4. Calculate the normalized low-pass and high-pass component values from Figs. 5 and 6, and denormalize them to the desired load resistances and crossover frequencies as described in Sections 11.1 and 11.2. Calculate the necessary bandpass parameters from Table 5 and find the normalized component values from the appropriate one of Figs. 7–10. Finally, denormalize to the radian center frequency $\omega_0 = 2\pi\sqrt{f_1 f_2}$ and the desired load resistance as described in Section 11.3.

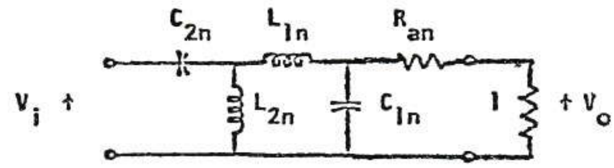


K=A

$C_n = K/R_0 \qquad R_{an} = K/H-1$

$L_n = 1/(KR_0) \qquad R_0 = 1 + R_{an}$

Fig. 7. First-order bandpass circuit and alignment formulas for the new crossovers. Calculate A, H parameters from first-order formulas in Table 5.



$C_{1n} = 1/(AR_0) \qquad L_{2n} = ER_0 \qquad K = B-1$

$L_{1n} = AR_0/K \qquad R_{an} = K/H-1 \qquad E = A-A/K$

$C_{2n} = K/(ER_0) \qquad R_0 = R_{an}+1$

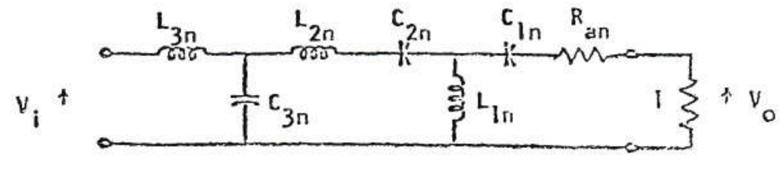
Fig. 8. Second-order bandpass circuits and alignment formulas for the new crossovers. Calculate A, B, H parameters from second-order formulas in Table 5.

13 CONCLUSION

Two-way all-pass crossovers are widely recommended because they offer a flat combined voltage magnitude response in a network that can be realized passively by simple ladder circuits. In addition a desirable phase response can be obtained with the even orders, and a flat combined power response with the odd orders. We have shown that there is no elementary way to implement three-way all-pass crossovers from these known two-way networks. In particular, neither cascading the two-way networks nor paralleling filters derived from them will do the job, even if the inevitable loading effects are ignored.

Because of this, we have derived new transfer function triples which do have the all-pass property. There are several forms which these functions might reasonably have taken. We picked the one that apparently requires the least amount of computation to obtain the functions and to align the relevant circuits. The form is that of a parallel filter crossover with a bandpass section obtained by cascading low- and high-pass filters derived directly from the low-pass section of the crossover. Also, the high-pass section is derived by a low-pass to high-pass frequency transformation from the low-pass section. The fact that the crucial low-pass section is an all-pole filter assures that the complete filter can be realized economically by simple resistance-terminated LC ladders.

These three-way networks have neither flat combined power in the odd orders nor the desirable phase behavior in the even orders, as do their two-way counterparts. Even so, the crucial all-pass response is preserved, and the aforementioned properties are satisfied in an approximate sense.



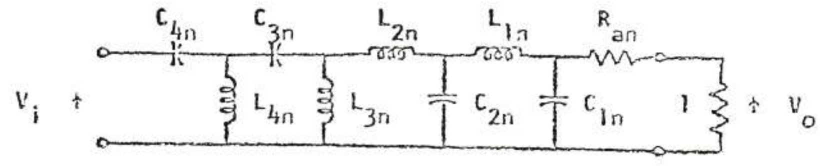
$C_{1n} = A/R_0 \qquad C_{3n} = F/(MR_0) \qquad E = B - C/A$

$L_{1n} = ER_0/A \qquad L_{3n} = MR_0/K \qquad F = A - A/E$

$C_{2n} = K/(ER_0) \qquad R_{an} = K/H - 1 \qquad K = C - A(B-1)/E$

$L_{2n} = R_0/F \qquad R_0 = 1 + R_{an} \qquad M = B-1-K/F_0 - EF/K$

Fig. 9. Third-order bandpass circuits and alignment formulas for the new crossovers. Calculate A, B, C, H parameters from third-order formulas in Table 5.



$C_{1n} = 1/(AR_0)$ $\quad C_{4n} = K/(R_0T)$ $\quad G = C-AF/E$

$L_{1n} = AR_0/E$ $\quad L_{4n} = TR_0/Q$ $\quad J = C-A(B-1)/E$

$C_{2n} = E/(GR_0)$ $\quad R_{an} = K/H-1$ $\quad K = F-JE/G$

$L_{2n} = GR_0/K$ $\quad R_0 = 1+R_{an}$ $\quad L = A-A/E$

$C_{3n} = Q/(PR_0)$ $\quad E = B-C/A$ $\quad N = J-GM/K$

$L_{3n} = PR_0$ $\quad F = D-C/A$ $\quad P = L-G/K$

$M = B-1-EL/G$ $\quad Q = M-N/P$

Fig. 10. Fourth-order bandpass circuits and alignments formulas for the new crossovers. Calculate A, B, C, D, H parameters from fourth-order formulas in Table 5.

Table 5. Bandpass circuit design parameters.

First order	$A = R + r$ $H = \|h\|R$ with parameter h from Eq. (28)
Second order	$A = a(R + r)$ $B = R^2 + a^2 + r^2$ $H = \|h\|R^2$ with parameters a, h from Eqs. (35) and (36), respectively
Third order	$A = bR + ar$ $B = aR^2 + ab + br^2$ $C = R^3 + a^2R + b^2r + r^3$ $H = \|h\|R^3$ with parameters a, b, h from Table 2 or 3.
Fourth order	$A = cR + ar$ $B = bR^2 + ac + br^2$ $C = aR^3 + abR + bcr + cr^3$ $D = R^4 + a^2R^2 + b^2 + c^2r^2 + r^4$ $H = \|h\|R^4$ with parameters a, b, c, h from Table 4.

These new three-way crossovers should be of significant value to the audio equipment designer in general and to the loudspeaker specialist in particular. We realize that the most effective loudspeaker crossover is arrived at by successive approximations of a specified overall acoustic output, but at the very least these new networks allow the approximation process to be started with a better initial configuration. Also, the new networks give the designer the assurance that any measured amplitude aberration is not caused by the crossover, but must arise from some other link in the system chain.

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APPENDIX
DERIVATION OF THIRD-ORDER EQUATIONS

The third-order all-pass derivation is started by substituting Eqs. (24)–(26) into Eq. (22) using Eq. (37) for $P(s)$. Letting $s = \omega j$, taking the magnitude squared, and clearing fractions gives

$$(\omega^6 - br^2\omega^4 + br^2\omega^2 - 1)^2 + (ar\omega^5 - (2r^3 + hR^3)\omega^3 + ar\omega)^2 = (\omega^6 - B\omega^4 + B\omega^2 - 1)^2 + (A\omega^5 - C\omega^3 + A\omega)^2 \quad (51)$$

where

$$A = bR + ar$$

$$B = aR^2 + ab + br^2$$

$$C = R^3 + a^2R + b^2r + r^3 .$$

Now the ω^2 coefficient on both sides of Eq. (51) must be the same. This gives

$$(a^2 - 2b)r^2 = (b^2 - 2a)R^2 + (a^2 - 2b)r^2 .$$

Since $R > 1$, it follows that

$$b^2 - 2a = 0 \quad (52)$$

which is equivalent to Eqs. (39) and (42) of Section 8.3. The equality of the ω^4 coefficients on both sides of Eq. (51) gives

$$-2arhR^3 + 2br^2 - 2ar^4 = (a^2 - 2b)R^4 . \quad (53)$$

Next, the fact that both sides of Eq. (51) must be equal when $\omega = 1$ yields

$$(2r^3 + hR^3 - 2ar)^2 = (R^3 + (a^2 - 2b)R + r^3)^2 . \quad (54)$$

Thus there are two cases. One of them results in

$$hR^3 = R^3 + (a^2 - 2b)R + 2ar - r^3 \quad (55)$$

which is formula (40) of Section 8.3 when solved for h. Substituting Eqs. (55) and (52) into Eq. (53) and rearranging in descending powers of b gives Eq. (38) in Section 8.3. These equations determine the first crossover.

The second case stemming from Eq. (54) gives

$$hR^3 = -R^3 - (a^2 - 2b)R + 2ar - 3r^3 \quad (56)$$

which is equivalent to Eq. (43) of Section 8.3. Finally, substituting Eqs. (56) and (52) into Eq. (53) gives Eq. (41) of Section 8.3 after appropriate rearrangement.

DERIVATION OF FOURTH-ORDER EQUATIONS

Eqs. (45)–(49) of Section 8.4 are derived here. When the polynomial (44) is substituted into Eqs. (24)–(26), these into Eq. (22), s is set equal to ωj, and the magnitude squared is taken, the result is

$$L(\omega) = R(\omega) \quad (57)$$

where

$$L(\omega) = (\omega^8 - br^2\omega^6 + (2r^4 + hR^4)\omega^4 - br^2\omega^2 + 1)^2 + (ar\omega^7 - cr^3\omega^5 + cr^3\omega^3 - ar\omega)^2$$

$$R(\omega) = (\omega^8 - B\omega^6 + D\omega^4 - B\omega^2 + 1)^2 + (A\omega^7 - C\omega^5 + C\omega^3 - B\omega)^2$$

$$A = cR + ar$$

$$B = bR^2 + ac + br^2$$

$$C = aR^3 + abR + bcr + cr^3$$

$$D = R^4 + a^2R^2 + b^2 + c^2r^2 + r^4 .$$

Equating the ω^{14} coefficients of $L(\omega)$ and $R(\omega)$ gives

$$(a^2 - 2b)r^2 = (c^2 - 2b)R^2 + (a^2 - 2b)r^2$$

which reduces to

$$c^2 - 2b = 0 \quad (58)$$

since $R \neq 0$. This is Eq. (45) of Section 8.4. Now equating ω^{12} coefficients in Eq. (57) gives

$$2R^4h + 4r^4 + (b^2 - 2ac)r^4 = (b^2 - 2ac + 2)(R^4 + r^4)$$

which simplifies to

$$R^8h = (b^2 - 2ac + 2)R^8/2 - 1 . \quad (59)$$

This is equivalent to Eq. (46) of Section 8.4. Equating ω^{10} coefficients and substitution of Eqs. (58) and (59) gives

$$(a^2 - 2b)R^{10} + (a^2 - b)(b^2 - 2ac + 2)R^6 + 2bR^2 - 2ac = 0$$

after a lengthy calculation. This is Eq. (47) of Section 8.4. Finally, letting $\omega = 1$ in Eq. (57) yields

$$(2 - 2br^2 + hR^4 + 2r^4)^2 = [R^4 + (a^2 - 2b)R^2 + (b^2 - 2ac + 2) + r^4]^2 .$$

Substitution of Eqs. (58) and (59) leads to either

$$(b^2 - 2ac)R^6 - 2(a^2 - 2b)R^4 - 2(b^2 - 2ac)R^2 - 4b = 0$$

or

$$(4 - 2ac + b^2)R^8 + 2(a^2 - 2b)R^6 + 2(b^2 - 2ac + 4)R^4 - 4bR^2 + r = 0 .$$

These are Eqs. (48) and (49), respectively, of Section 8.4.

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Robert M. Bullock, III, received his B.S., M.A., and Ph.D. degrees in mathematics from the University of Cincinnati in 1961, 1962, and 1966, respectively. Since then he has been in the department of mathematics and statistics, Miami University, Oxford, Ohio, and is currently an associate professor. His mathematical specialty is differential equations, but his current work is concerned with linear models of loudspeaker systems.

Dr. Bullock is a member of the Audio Engineering Society, Sigma Xi, and the Society for Industrial and Applied Mathemtics. He is also a contributing editor of *Speaker Builder Magazine*.

CORRECTIONS TO "PASSIVE THREE-WAY ALL-PASS CROSSOVER NETWORKS."

I have discovered three errors in the above paper.[1] The following corrections should be made: On page 636, Fig. 7 change the formula $L_n = 1/(KR_0)$ to $L_n = R_0/K$. On page 636, Fig. 9 change the formula $M = B - 1 - K/F_0 - EF/K$ to $M = B - 1 - K/F - EF/K$. On page 637, Fig. 10 add the formula $T = N - PK/Q$.

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