Loudspeakers in Vented Boxes: Part I*

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An investigation of the equivalent circuits of loudspeakers in vented boxes shows that it is possible to make the low-frequency acoustic response equivalent to an ideal high-pass filter or as close an approximation as is desired. The simplifying assumptions appear justified in practice and the techniques involved are simple.

The low-frequency performance of a loudspeaker can be adequately defined by three parameters, the resonant frequency f_s, a volume of air V_{as}, equivalent to its acoustic compliance, and the ratio of electrical resistance to motional reactance at the resonant frequency Q_e. From these three parameters, the electroacoustic efficiency η can be found also. A plea is made to loudspeaker manufacturers to publish these parameters as basic information on their product. The influence of other speaker constants on these parameters is investigated.

When f_s and V_{as} are known, a loudspeaker box can be designed to give a variety of predictable responses which are different kinds of high-pass 24-dB per octave filters. For each response, a certain value of Q is required which depends not only on the Q_e of the loudspeaker but also the damping factor of the amplifier, for which a negative value is often required.

The usual tuning arrangement leads to a response which can be that of a fourth-order Butterworth filter. This, however, is only a special case, and a whole family of responses may be obtained by varying the volume and tuning of the box. Also an empirical "law" is observed that for a given loudspeaker the cutoff frequency depends closely on the inverse square root of the box volume. The limitations of this "law" may be overcome by the use of filtering in the associated amplifier. For example, for a given frequency response, the box volume can be reduced at the price of increased low-frequency output from the amplifier and vice versa, with little change in the motion required of the loudspeaker.

Acoustic damping of the vent is shown to be unnecessary. Examples are given of typical parameters and enclosure designs.

Editor's Note: The theory of vented-box or bass-reflex loudspeaker baffles has always seemed to have an air of mystery, probably because the total electroacoustic system has four degrees of freedom and seems four times as complicated as the closed-box baffle with its two degrees of freedom. Beranek gives a good foundation for theoretical analysis and Novak has performed numerous valuable calculations. Those working in the design of loudspeakers have used these analysis techniques and probably asked essentially the same seven questions that A. N. Thiele recognized at the turn of the previous decade.

The seven questions and their answers were published in the August 1961 issue of the *Proceedings of the IRE Australia*, and the elegance of the answers adequately justifies republication of Thiele's work in the *Journal of the Audio Engineering Society*. In his classic discourse Thiele observes that the topology of the equivalent circuit (Fig. 1) is simply that of a high-pass filter. If suffi-

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cient simplification can be justified, Thiele reasons that the methods of modern network synthesis should be applicable to loudspeakers. This is a profound observation because it means that once the system transfer function is chosen, a logical sequence can be followed to specify driver and baffle parameters. This is much more efficient than the cut and try methods based on either analysis or measurements.

Although the idea is profound because of its simplicity, much work is required to develop, utilize, and demonstrate its use. In the interest of compatibility with format in this Journal, we have received permission from A. N. Thiele to republish his work in two parts. This first part develops the synthesis approach and summarizes all of vented-box design in a table of 28 alignments. The second part will apply the method and draw some very pertinent conclusions about efficiency, driver Q, box volume, and amplifier output impedance.

The high point of this work is Table I which gives 28 alignments for vented-box loudspeakers. I have been so impressed with this table that I have written a Fortran program to quickly apply Thiele's synthesis methods to any loudspeaker with adequately known parameters. This program and a run or two for typical woofers will be published after Part II.

In considering this manuscript for republication, Thiele has suggested that after 10 years his only change of attitude would be to change the emphasis in Section XIV (Part II). In contrast to the original preference for use of a closed box (which is still quite valid), Thiele would now emphasize the use of a vented box for measurements. This is indeed a trifling matter and in concurring with Thiele's opinion, I can only add emphasis to how well this paper has passed the test of time—it is just as pertinent now as it was ten years ago.

J. R. Ashley

I. INTRODUCTION: The technique of using a vented box to obtain adequate low-frequency response from a loudspeaker has been known for many years. The principle seems simple, yet the results obtained are variable. Since comparatively cheap and reliable methods of acoustic measurement, especially at low frequencies, virtually do not exist, the only check of results is the "listening test." The listening test is after all the final criterion of the performance of an electroacoustic system, but as a method of adjusting for optimum it is very poor indeed. Quite apart from one's prejudices and memories of previous "acceptable" equipments, the adjustment of a vented box in ignorance of the loudspeaker parameters involves two simultaneous adjustments, box tuning and amplifier damping. And again there is a strong temptation to adjust the low-frequency response to something other than flat to "balance" response errors at high frequencies, when in fact the two problems should be tackled separately.

For a long time it has seemed to the writer that the methods of design of vented boxes were unsatisfactory, leaving a number of questions unanswered.

1) What size of box should be chosen? Usually it seems the larger the better, but how much better is a large box and what penalty does one pay for a small box? And for a given speaker, what *is* a "large" box or a "small" box?

2) What amplifier damping should be used? In general the answer is, the heavier the damping the better, though with high-efficiency speakers this could cause a loss of low frequencies. But then again, negative damping is sometimes used, especially in the United States. And when vented enclosures often give excellent results, why should they be known by some as "boom boxes"?

3) Is it advisable or necessary to use acoustic damping to flatten the response? Some claim good results [1] while others [2] warn against it. The general principle of flattening response with parasitic resistance, and thus dissipating hard-won power, seems wrong, especially in an output stage and when a maximum bandwidth is sought. The principle seems to apply equally to an amplifier–loudspeaker–box combination and a video output stage.

4) To what frequency should the vent be tuned? The conventional answer is to tune it to the loudspeaker resonant frequency, but Beranek [3, p. 254] mentions that "for a very large enclosure, it is permissible to tune the port to a frequency below the loudspeaker resonance," while small boxes are sometimes tuned above loudspeaker resonance.

5) What should be the area of the vent? The conventional answer is to make it equal to the piston area of the loudspeaker, but Novak [2] states that "it is permissible to use any value of vent area," and again "the vent area should not be allowed to be less than 4 in^2." Again, should we use only a hole for the vent or should we use a duct or tunnel?

6) If we equalize the amplifier to correct deficiencies in the speaker and enclosure, what penalties result for example in distortion? Can we trade amplifier size for box size?

7) Assuming that we know how to design a box (and associated amplifier) given the loudspeaker parameters, how may the parameters be measured?

There are other questions that could be asked but the seven above seem the most important; at any rate, they are the ones that the present paper hopes to answer.

II. DERIVATION OF BASIC THEORY

The theory of operation of loudspeakers in vented boxes has been covered so many times in the literature [3, pp. 208-258], [4] that it should be unnecessary to repeat it here; therefore only sufficient of the theory will be quoted to make the present approach intelligible.

This approach derives from Novak [2] to whom the reader is referred, not only for his method, but for his introductory paragraph . . . "Trade journals tell of 'all new enclosures, revolutionary concepts, and totally new principles of acoustics' when in reality there is a close identity with enclosure systems described long ago in well-known classics on acoustics." This should be framed and hung on the audio engineer's wall alongside Lord Kelvin's dictum. The present paper is the result of a different emphasis on, and interpretation of, Novak's treatment. It should be emphasized that, unless stated specifically otherwise, the results apply only to the "piston range" of the speaker. This is the region where the circumference of the speaker is less than the wavelength of radiated sound, i.e., below 400 Hz for a 12-inch speaker, and below 1 kHz for a 5-inch speaker. The performance of loudspeakers above the piston range is another subject altogether.

We will be dealing later with a simplified equivalent

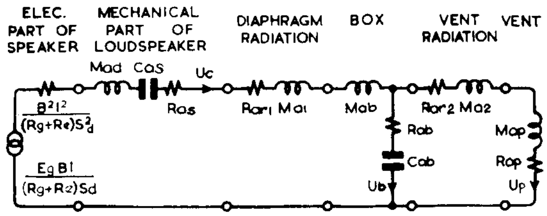

Fig. 1. Complete (electromechanical) acoustical circuit of loudspeaker in vented box (after Beranek [3]).

circuit, but first consider Fig. 1 in which the complete equivalent circuit of the loudspeaker and enclosure is given in acoustical terms.

We note that there are three possible equivalent circuits, electrical, mechanical, and acoustical. To convert from electrical to mechanical units,

$$Z_m = B^2l^2/Z_e \tag{1}$$

where

- Z_e electrical impedance
- Z_m equivalent mechanical impedance
- B magnetic flux density in air gap
- l length of wire in air gap.

Again to convert from mechanical to acoustic units,

$$Z_a = Z_m/S_d{}^2 \tag{2}$$

where

- Z_a acoustical impedance
- S_d equivalent piston area of diaphragm (usually taken as area inside first corrugation).

Taking then in Fig. 1 the first impedance after the generator which is the acoustical equivalent of the electrical resistance of the amplifier output impedance R_g in series with the voice coil resistance R_e, we can see that the various equivalents for this impedance are

$$Z_e = R_g + R_e \tag{3}$$
$$Z_m = B^2l^2/(R_g+R_e) \tag{4}$$
$$Z_a = B^2l^2/S_d{}^2(R_g+R_e). \tag{5}$$

In Fig. 1,

- E_g open-circuit voltage of audio amplifier
- M_{ad} $(= M_{md}/S_d{}^2)$ acoustic mass of diaphragm and voice coil
- M_{md} mechanical mass as usually measured
- C_{as} acoustic compliance of suspension
- R_{as} acoustic resistance of suspension
- R_{ar1} acoustic radiation resistance for front side of loudspeaker diaphragm
- M_{a1} acoustic radiation mass (air load) for front side of loudspeaker diaphragm
- M_{ab} acoustic mass of air load on rear side of loudspeaker
- R_{ab} acoustic resistance of box
- C_{ab} acoustic compliance of box
- R_{ar2} acoustic radiation resistance of vent
- M_{a2} acoustic radiation mass (air load) of vent
- M_{ap} acoustic mass of air in vent
- R_{ap} acoustic resistance of air in vent
- U_c volume velocity of cone
- U_b volume velocity of box
- U_p volume velocity of port, or vent.

The advantage of using this large complete equivalent circuit in the first place is that the equivalent circuit of the loudspeaker in a totally enclosed box may be shown by removing the mesh representing the vent. To represent the speaker operated in an infinite baffle, C_{ab} and R_{ab} are short-circuited. If the speaker is operated in open air (unbaffled), the circuit is as in an infinite baffle, but the values of R_{ar1} and M_{a1} are modified [see 4, Fig. 5.2]. The details of these circuits are very well covered in [3] from which Fig. 1 and the accompanying symbols are taken.

To make the circuit more manageable, we simplify it to Fig. 2.

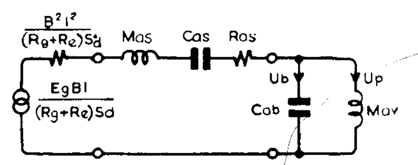

Fig. 2. Simplified acoustical circuit of loudspeaker in vented box.

1) The three acoustic masses M_{ad}, M_{a1}, and M_{ab} are lumped together to make a single mass M_{as}. However, we must be careful to remember that this is an artifice. M_{as} is not fixed, and some error results by assuming it to be so. For example, the reduction of M_{ab} and hence of M_{as} when the speaker is tested in open air causes a rise in resonant frequency, which must be accounted for in measurements, as in Section XIV.

2) R_{ar1} and R_{ar2} are neglected in the equivalent circuit, even though they are responsible for the acoustic output of the loudspeaker. The whole essence of Novak's theoretical model which makes a simple solution possible is that a loudspeaker is a most inefficient device. In measurements of fifty loudspeakers using the method of Section XIV covering a wide range of sizes and qualities, efficiencies ranged between 0.4% and 4%. For this reason, the radiation resistances may be safely neglected. Since radiation resistance varies with frequency squared, this simplifies analysis considerably. For, as pointed out in [3, p. 216], the radiation resistance of a loudspeaker in a "medium-sized box (less than 8 ft³)" is approximately the radiation impedance for a piston in the end of a long tube. And the radiation resistance of the vent (or port) is the same. Thus

$$R_{ar1} = R_{ar2} = \pi f^2 \rho_o / c \tag{6}$$

where ρ_o is the density of air and c is the velocity of sound in air.

Note that the radiation resistance is independent of the dimensions of the piston or vent. Note also that Eq. (6) is an approximation which is accurate only in the piston range of the loudspeaker (compare [3, Fig. 5.7] or [4, Fig. 5.2]).

3) M_{a2} and M_{ap} are lumped together as M_{av}, the total air mass of the vent.

4) R_{ab} and R_{ap} are neglected since for most practical purposes their Q is very high compared with that of the loudspeaker, especially when its damping is properly controlled by the amplifier.

For example, it will be shown later that the Q of speak-

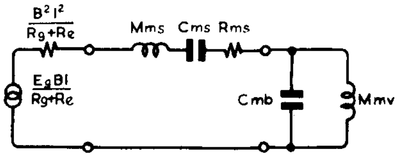

Fig. 3. Simplified mechanical circuit of loudspeaker in vented box.

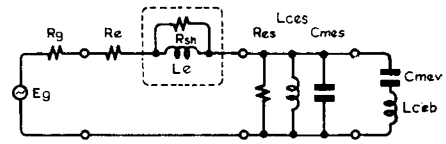

Fig. 4. Simplified electrical circuit of loudspeaker in vented box.

er plus amplifier for a vented box will usually lie between 0.3 and 0.5. The Q of the vent, on the other hand, can be found by combining [3, Eqs. (5.54) and (5.55)] to give

$$Q_v = \omega M_{ap}/R_{ap} = (S_v f/\mu)^{1/2}(l'+1.70a)/(l'+2a) \qquad (7)$$

where

- Q_v effective Q of vent
- S_v area of vent (assumed to have constant cross section)
- l' actual length of vent
- a effective radius of vent
- μ kinematic coefficient of viscosity; for air at NTP, $\mu = 1.56 \times 10^{-5}$ m²/s.

Thus if $S_v = 4$ in², the bottom limit specified by Novak, and $f = 25$ Hz, then $Q_v = 64$.

Since these are the smallest values of S_v and f likely to be found in practice, it is clear that little error will result from this source, and this is confirmed in Section XI. In the preceding discussion, the effect of M_{a2} and R_{ar2} has been neglected, but in no case investigated has the total Q_v fallen below 30.

5) As a result of measurements of fifty loudspeakers, it appears that the Q_a of the speaker due to R_{as} lies usually between 3 and 10, so that this does not affect matters greatly, but since R_{as} can be lumped with the equivalent electrical resistance (see Eq. (8)) and because it has some importance in the loudspeaker measurements of Section XIV, it is included in Fig. 2

The mechanical equivalent circuit (Fig. 3) is derived from Fig. 2 by multiplying all the acoustical impedances by the conversion factor S_d^2 as in Eq. (2). Thus these impedances represent the mechanical impedances at the loudspeaker diaphragm due to the whole acoustical–mechanical circuit. Since the conversion is obtained by multiplying by a constant, the form of the circuit remains the same. However, when the conversion is made from Fig. 3 to Fig. 4, the electrical equivalent circuit, it can be seen from Eq. (1) that an impedance inversion takes place. Thus all series elements become parallel elements, inductances become capacitances, and vice versa. Thus L_{ces} is the electrical inductance due to the compliance of the loudspeaker suspension, C_{mes} is the electrical capacitance due to the mass of the loudspeaker cone, C_{mev} is the electrical capacitance due to the mass of the vent, and L_{ceb} is the electrical inductance due to the compliance of the box. In Fig. 4 an additional pair of circuit elements which were neglected in the earlier circuits have been added within the dashed lines. These are the inductance and shunt resistance (largely due to eddy current loss in the pole piece and front plate) of the voice coil.

It is hoped that this will not cause confusion. These elements contribute very small effects at the low frequencies we are considering, but show the reason for the shape of the resulting electrical impedance curve of Fig. 5 above f_n. However, this will be of greater importance when we come to testing procedures in Section XIV.

III. DERIVATION OF RESPONSE CURVE

The expression for the frequency response of the system is obtained by analysing the circuit of Fig. 2. To simplify the expression, we lump all the series resistance into a total acoustic resistance,

$$R_{at} = R_{as} + [B^2l^2/(R_g+R_e)S_d^2]. \qquad (8)$$

Now we have seen already that the radiation resistances of speaker and vent must always be the same. And since the radiated sound depends on the sum of the volume velocities U_c and U_p (or rather their difference, since U_p derives from the back pressure of the speaker), then the acoustic power output is

$$W_{ao} = |U_c - U_p|^2 R_{ar1} \qquad (9)$$

while the nominal electrical input power is

$$W_{ei} = E_g^2 R_e/(R_g+R_e)^2. \qquad (10)$$

Thus the efficiency is

$$\eta = W_{ao}/W_{ei} = [|U_c - U_p|^2 R_{ar1}(R_g+R_e)^2]/(E_g^2 R_e). \qquad (11)$$

Analyzing the circuit, we find that

$$(U_c - U_p)/[E_g Bl/S_d(R_g+R_e)] = 1/pM_{as} \times \left[\frac{p^4 M_{as}M_{av}C_{as}C_{ab}}{\left\{\begin{array}{c} p^4 M_{as}M_{av}C_{as}C_{ab} + p^3 M_{av}C_{as}C_{ab}R_{at} \\ + p^2(M_{as}C_{as}+M_{av}C_{as}+M_{av}C_{ab}) + pC_{as}R_{at} + 1 \end{array}\right\}}\right]. \qquad (12)$$

To make the expression easier to manage we write $E(p)$ for the expression inside the square bracket on the right-hand side which is a fourth-order high-pass filtering function. Also if $j\omega$ is written for p, the steady-state response $E(j\omega)$ *is* found. We also convert pM_{as} from the operational form to the steady-state form $j\omega M_{as}$, and then substitute

$$M_{ms} = M_{as}S_d^2. \qquad (13)$$

This puts the expression for mass into a more intelligi-

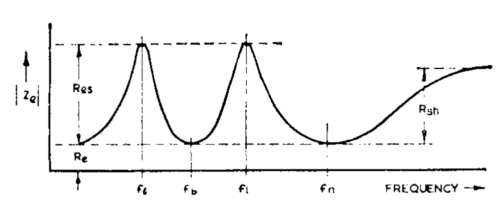

Fig. 5. Typical impedance curve of loudspeaker in vented box.

ble form, but it is emphasized that the total loudspeaker mechanical mass M_{ms} includes not only the mass of the cone plus voice coil, but also the mechanical equivalent of the acoustic air load. The latter is only a small part of the total, but varies with the speaker's environment, e.g., box volume [3]. Thus if we substitute Eqs. (6), (12), and (13) in Eq. (11),

$$\eta = \rho_o B^2 l^2 S_d^2 |E(j\omega)|^2 / 4\pi c R_e M_{ms}^2 \tag{14}$$

or

$$\eta = (\rho_o/4\pi c)(B^2 l^2 S_d^2 / R_e M_{ms}^2) |E(j\omega)|^2. \tag{15}$$

Thus the expression for efficiency contains three parts:
1) a constant part containing physical constants,
2) a constant part containing speaker parameters,
3) a part $|E(j\omega)|^2$ which varies with frequency.

IV. CONTROLLING THE FREQUENCY RESPONSE

The problem of greatest interest is the control of frequency response; so we consider first (3), $|E(j\omega)|^2$, or preferably its operational form $E(p)$. To make this easier to manage we substitute in $E(p)$ of Eq. (12)

$$T_s^2 = (1/\omega_s)^2 = M_{as}C_{as} \tag{16}$$
$$T_b^2 = (1/\omega_b)^2 = M_{av}C_{ab} \tag{17}$$
$$Q_t = (M_{as}/C_{as})^{1/2}/R_{at} \tag{18}$$

where ω_s is the resonant frequency. ω_b is the box resonant frequency, or more exactly, the frequency at which the acoustic mass of the vent resonates with the acoustic capacitance of the box. It should not be confused, as is often done, with f_h or f_l of Fig. 5, which are by-products of f_s and f_b (see Eqs. (105) and (106)).

Q_t is the total Q of the loudspeaker when connected to its amplifier. The acoustic resistance in the loudspeaker R_{as} has a small effect, but usually the resistances reflected from the loudspeaker resistance R_e *and the amplifier* R_g contribute the greater part of Q_t. Then $E(p)$ of Eq. (12) becomes

$$E(p) = \frac{p^4 T_b^2 T_s^2}{\left\{ \begin{array}{c} p^4 T_b^2 T_s^2 + p^3 (T_b^2 T_s / Q_t) \\ + p^2 [T_b^2 + T_s^2 + T_b^2 C_{as}/C_{ab}] + p(T_s/Q_t) + 1 \end{array} \right\}} \tag{19}$$

For many purposes this is more conveniently written as

$$E(p) = 1/\{1 + 1/pQ_t T_s + (1/p^2)[1/T_b^2 + 1/T_s^2 + C_{as}/C_{ab}T_s^2] + 1/p^3 T_b^2 T_s Q_t + 1/p^4 T_b^2 T_s^2\}. \tag{20}$$

This expression corresponds to Novak's expression for the modulus in his Eq. (15) which is simplified into his Eq. (16). (Note that in the captions for his Figs. 7, 9, 11, 12, and 13, a positive sign is wrongly substituted for a negative sign).

As stated before, this is a fourth-order high-pass function, that is, it has an asymptotic slope in the attenuation band of 24 dB per octave, and can be written in the general form

$$E(p) = 1/\{1 + x_1/pT_0 + x_2/p^2 T_0^2 + x_3/p^3 T_0^3 + 1/p^4 T_0^4\} \tag{21}$$

which is defined by a time constant T_0 ($= 1/\omega_0$, the nominal cutoff frequency) and three coefficients x_1, x_2, x_3 which determine the shape of the response curve. In fact, the general expression is often written with a constant x_0 and x_4 instead of the two unity coefficients in the denominator of Eq. (21); but the expression can always be reduced to the form of Eq. (21) by division of the whole expression by a constant, and suitable adjustment of T_0 and the x coefficients. Considering Eq. (20) now from the viewpoint of what can be done with a given speaker, the parameters C_{as} and T_s are fixed. Thus there are three variables Q_t, T_b, and C_{ab}, and it is possible to achieve any desired shape of curve (i.e., any desired combination of the three x coefficients); but in doing so T_0 is determined (see Eq. (27)).

For identity between the two Eqs. (20) and (21), the coefficients of the various powers of p must be identical, that is,

$$x_1/T_0 = 1/Q_t T_s \tag{22}$$
$$x_2/T_0^2 = 1/T_b^2 + 1/T_s^2 + C_{as}/C_{ab}T_s^2 \tag{23}$$
$$x_3/T_0^3 = 1/Q_t T_b^2 T_s \tag{24}$$
$$1/T_0^4 = 1/T_b^2 T_s^2. \tag{25}$$

From these, the relationships can be established

$$T_b/T_s = x_1/x_3 \tag{26}$$
$$T_0/T_s = (x_1/x_3)^{1/2} \tag{27}$$
$$Q_t = 1/(x_1 x_3)^{1/2} \tag{28}$$
$$C_{as}/C_{ab} = (x_1 x_2 x_3 - x_3^2 - x_1^2)/x_1^2. \tag{29}$$

The Hurwitz criteria [5] for stability of a network defined by Eq. (21) are
1) all the x coefficients are positive,
2) $x_1 x_2 x_3 - x_3^2 - x_1^2$ is positive.

If (1) and (2) are true, then all the parameters determined by the four Eqs. (26)–(29) are positive and therefore realizable. Thus we have in the four equations a set of simple relationships which enable us to achieve, for any speaker, any shape of low-frequency cutoff (fourth-order) characteristic. The only requirement is that we have sufficient freedom to choose a suitable box resonant frequency $1/T_b$, box volume C_{ab}, and total Q of speaker plus amplifier Q_t, and can accept the resulting value of T_0.

The first parameter T_b presents no practical difficulty; the second, C_{ab}, can cause trouble if space is limited, but in this case, as shown in Section VII, we can work backward and choose a suitable response characteristic to suit the box size; the third, Q_t, is controlled by the source impedance of the amplifier. If the required Q_t is greater than the speaker's natural Q, a positive output impedance will be required of the amplifier and this can be controlled by the usual negative feedback techniques. If less, a negative output impedance will be required, and this can be achieved by applying feedback from a separate winding on the voice coil, or by a combination of positive current and negative voltage feedback. There is a practical limit here if the degree of negative impedance required is too large, but this will be discussed in Section XII.

V. SOME PRACTICAL RESPONSE CURVE SHAPES

Fourth-Order Butterworth Response

Armed with Eqs. (26)–(29) we can calculate the parameters required for different response characteristics. The most obvious one to try first is the fourth-order maximally flat (Butterworth)[1] characteristic for which

$$|E(j\omega)| = 1/[1+(\omega_o/\omega)^8]^{1/2} \tag{30}$$

or

$$|E(j\omega)|^2 = 1/[1+(\omega_o/\omega)^8] \tag{31}$$

and, in the operational form,

$$E(p) = 1/(1+2.613/pT_o+3.414/p^2T_o^2 +2.613/p^3T_o^3+1/p^4T_o^4). \tag{32}$$

Note that in Eq. (31) and others which will follow, the ratio of any two frequencies, say ω_a/ω_b, is identical to f_a/f_b. Note also that all Butterworth responses are 3 dB down when $\omega = \omega_o$, i.e., $\omega T_o = 1$.

A characteristic of Butterworth responses, though not peculiar to them, which simplifies calculations even further is that in all cases

$$x_1 = x_3. \tag{33}$$

Thus in this class or response,

$$T_b = T_s \tag{34}$$
$$T_o = T_s \tag{35}$$
$$Q_t = 1/x_1 \tag{36}$$
$$C_{as}/C_{ab} = x_2-2. \tag{37}$$

Thus in the fourth-order case where

$$x_1 = x_3 = 2.613 \tag{38}$$
$$x_2 = 3.414 \tag{39}$$

we have

$$Q_t = 0.383 \tag{40}$$
$$C_{as}/C_{ab} = 1.414. \tag{41}$$

This is alignment no. 5 of Table I. The term "alignment" seems appropriate since the problem is similar to the choice of alignments for other filters, e.g., RF and IF amplifiers. This is obviously the conventional type of box alignment, for the box frequency f_b is identical with the speaker resonant frequency f_s, and also the frequency f_3 with which the response is -3 dB. Note that because of the rapid change of attenuation the response is only -0.9 dB at $1.2f_s$.

However, it also shows that a true maximally flat characteristic is obtained only if the correct values of box size C_{as} and especially Q_t are chosen also. It is easy to show from Eq. (20) that in any alignment, at the upper resonant frequency (f_h of Fig. 5), the response is

$$E(j\omega) = j(Q_t\omega_h/\omega_s)/[1-(\omega_b^2/\omega_h^2)] \tag{42}$$

that is, the response varies directly with Q_t. Also at the box resonant frequency, f_b

$$E(j\omega) = (C_{ab}/C_{as})(\omega_b^2/\omega_s^2) \tag{43}$$

that is, the response is independent of Q_t. (The response at f_l is similar to Eq. (42) when ω_h is replaced by ω_l, but as this is in the attenuation band, it is less important.) Thus if Q_t is twice the optimum value, there will be a response peak 6 dB high. Now as a general rule a speaker with a Q of about 0.4, as required in this case, is usually of high quality.

A Q of 0.8 is typical of a medium quality speaker and a Q of 1.6 is typical of a low ("popular" or "skimped-magnet") quality speaker. Thus these speakers would have response peaks (at $1.76\omega_s$ in this case) of 6 dB and 12 dB, respectively, if fed from a zero output impedance amplifier, 12 dB and 18 dB if fed from an amplifier with impedance equal to loudspeaker resistance R_e (e.g., pentode with 6-dB negative voltage feedback), and even more with higher amplifier impedances. Hence the expression "boom box."

An amplifier with negative output impedance half that of the loudspeaker resistance R_e, a quite feasible figure, would correct the medium quality speaker, and reduce the peak on the cheaper one to 6 dB. An amplifier with a negative output impedance three quarters of R_e, to correct the cheaper speaker, is possible but would need care in respect of stability (see Section XII).

[1] Hence the expression Butterworth box. However, in spite of the phonetic similarity, butter boxes are not in general suitable as loudspeaker enclosures.

Fifth-Order Butterworth Response

This has the characteristic

$$|E(j\omega)|^2 = 1/[1+(\omega_o/\omega)^{10}]. \tag{44}$$

The operational form can be factorized to

$$E(p) = 1/[(1+1/pT_o)(1+\sqrt{5}/pT_o +3/p^2T_o^2+\sqrt{5}/p^3T_o^3+1/p^4T_o^4)] \tag{45}$$

which is the characteristic of two filters in cascade: 1) a first-order filter which can be provided by a CR network with a time constant T_o, and 2) a fourth-order filter provided by a loudspeaker and box for which

$$T_o = T_s = T_b \tag{46}$$
$$Q_t = 0.447 \tag{47}$$
$$C_{as}/C_{ab} = 1. \tag{48}$$

The alignment, no. 10 of Table I, has the advantage if the extra box size can be tolerated (a smaller value of C_{as}/C_{ab} means a larger box) that a maximally flat response can be obtained down to the loudspeaker resonant frequency, while at the same time, a very simple "rumble" filter tapers off the input to the amplifier in the attenuband. This helps the amplifier, but more importantly it greatly reduces the maximum flux density in the output transformer and also the maximum excursion of the loudspeaker (see Section X and Fig. 10).

Sixth-Order Butterworth Response

This has the characteristic

$$|E(j\omega)|^2 = 1/[1+(\omega_o/\omega)^{12}] \tag{49}$$

while the operational form may be factorized to

$$E(p) = 1/[(1+1.932/pT_o+1/p^2T_o^2)(1+1.414/pT_o+1/p^2T_o^2)(1+0.518/pT_o+1/p^2T_o^2)]. \tag{50}$$

As in the previous case, the overall alignment is achieved by providing one factor with an external filter, in this case second order, and making the fourth-order response of the loudspeaker plus box the product of the two remaining factors. Thus we can obtain the identical response in three different ways. These are listed in Table I as alignments no. 15, 20, and 26, the three separate classes depending on whether the auxiliary electrical circuit has the lowest, middle, or highest x value of the three factors in the alignment. Not only do the three alignments produce the same response, but as shown later (Section X and Fig. 10) the cone excursions are identical.

	Alignment Details				Box Design				Auxiliary Circuits				Approximately Constant Quantities	
	No.	Type	k	Ripple (db)	f_3/f_s	f_3/f_b	C_{as}/C_{ab}	Q_t	f_{aux}/f_3	y_{aux}	Peak Lift (db)	f_{pk}/f_3	$\frac{C_{as}f_s^2}{C_{ab}f_3^2}$	$\frac{Q_t f_b}{f_s}$
Quasi-Third Order	1	QB_3	—	—	2.68	1.34	10.48	.180	—	—	—	—	1.47	.360
	2	QB_3	—	—	2.28	1.32	7.48	.209	—	—	—	—	1.44	.362
	3	QB_3	—	—	1.77	1.25	4.46	.259	—	—	—	—	1.43	.367
	4	QB_3	—	—	1.45	1.18	2.95	.303	—	—	—	—	1.41	.371
Fourth Order	5	B_4	1.0	—	1.000	1.000	1.414	.383	—	—	—	—	1.41	.383
	6	C_4	.8	—	.867	.935	1.055	.415	—	—	—	—	1.41	.384
	7	C_4	.6	0.2	.729	.879	.729	.466	—	—	—	—	1.37	.386
	8	C_4	—	0.9	.641	.847	.559	.518	—	—	—	—	1.36	.392
	9	C_4	—	1.8	.600	.838	.485	.557	—	—	—	—	1.35	.398
Fifth Order	10	B_5	1.0	—	1.000	1.000	1.000	.447	1.00	—	—	—	—	—
	11	C_5	.7	—	.852	.934	.583	.545	1.43	—	—	—	—	—
	12	C_5	.4	0.25	.724	.889	.273	.810	2.50	—	—	—	—	—
	13	C_5	.355	0.5	.704	.882	.227	.924	2.93	—	—	—	—	—
	14	C_5	.278	1.0	.685	.877	.191	1.102	3.60	—	—	—	—	—
Sixth Order Class I	15	B_6	1.0	—	1.000	1.000	2.73	.299	1.00	−1.732	+6.0	1.07	—	—
	16	C_6	.8	—	.850	.868	2.33	.317	1.01	−1.824	+7.7	1.06	—	—
	17	C_6	.6	—	.698	.750	1.81	.348	1.02	−1.899	+10.1	1.05	—	—
	18	C_6	.5	—	.620	.698	1.51	.371	1.03	−1.930	+11.6	1.05	—	—
	19	C_6	.414	0.1	.554	.659	1.25	.399	1.04	−1.951	+13.2	1.04	—	—
Sixth Order Class II	20	B_6	1.0	—	1.000	1.000	1.000	.408	1.00	0	—	—	—	—
	21	C_6	.8	—	.844	.954	.722	.431	1.10	−.438	+0.2	2.36	—	—
	22	C_6	.6	—	.677	.917	.500	.461	1.21	−.941	+1.1	1.77	—	—
	23	C_6	.5	—	.592	.902	.414	.484	1.27	−1.200	+1.9	1.63	—	—
	24	C_6	.414	0.1	.520	.890	.353	.513	1.31	−1.414	+3.0	1.55	—	—
	25	C_6	.268	0.6	.404	.876	.276	.616	1.37	−1.732	+6.0	1.47	—	—
Sixth Order Class III	26	B_6	1.0	—	1.000	1.000	.732	.518	1.00	+1.732	—	—	—	—
	27	C_6	.268	0.6	.778	.911	.110	1.503	2.73	0	—	—	—	—
	28	QB_3	—	—	.952	.980	1.89	.328	1.08 mean	—	6.0	0	—	—

Table I. Summary of loudspeaker alignments.

This illustrates a general principle that box size can be exchanged for amplifier power. The only additional penalties are as follows:

1) additional heating of the voice coil by signals in the region of the cutoff frequency, and
2) the requirement of a smaller value of Q_t as the box volume is decreased.

The performance required of the auxiliary filtering is given in the last four columns of Table I, whose terms are illustrated in Fig. 6. Instead of the parameter x in the expression

$$E(p) = 1/(1+x/pT_o+1/p^2T_o^2) \tag{51}$$

the response shapes are defined in Table I by the parameter y in the expression

$$|E(j\omega)|^2 = 1/[1+y(\omega_o/\omega)^2+(\omega_o/\omega)^4] \tag{52}$$

where

$$y = x^2-2 \tag{53}$$

as given in a previous paper [6]. When y is zero or positive there is no peak in the response as shown in Fig. 6, but when y is negative there is a peak whose frequency and amplitude are given in Table I. The amplitude of response at the nominal cutoff frequency f_{aux} of this auxiliary filter is given by

$$|E(j\omega)| = 1/(2+y)^{1/2}. \tag{54}$$

Chebyshev Responses

If the real values of the poles of a Butterworth function are all multiplied by the same factor k, which is less than one, a Chebyshev or "equal ripple" function results [7]. Chebyshev filters are characterized by a flat response in the passband except for ripples which are equal in amplitude, (see curve 8 of Fig. 8). Beyond cutoff, the response falls at a rate whose maximum is greater than the asymptotic slope. Typical values are tabulated in Table I with the type names C_4, C_5, and C_6 representing Chebyshev responses of fourth, fifth, and sixth order. It will be seen from the table that a considerable change in alignment occurs before the ripples become serious in magnitude. For our purpose here, the Chebyshev responses provide a means of carrying the useful response of the speaker plus box combination well below the speaker resonant frequency f_s (which is also cutoff frequency f_o in the Butterworth cases). This is done by tuning the box to below f_s, but not as low as the cutoff frequency (defined here as f_3, the frequency where the response is 3 dB down). The box size C_{ab} is increased, and to some extent, so is Q_t.

The increase in useful low-frequency response is considerable. In alignment no. 9, a response down to $0.6f_s$ is obtainable without amplifier assistance, if a ripple of 1.8 dB can be tolerated. In alignment no. 25, where a maximum lift of 6 dB is required from the amplifier before its response falls off, a flat response can be obtained down to nearly $0.4f_s$.

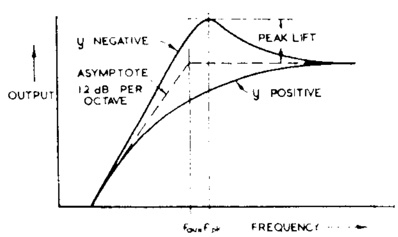

Fig. 6. Typical curves for second-order auxiliary filter, illustrating terms used in Table I.

Quasi-Butterworth Third-Order Responses

This long name disguises a class of responses characterized by

$$|E(j\omega)|^2 = 1/[1+y_3(\omega_o/\omega)^6+y_4(\omega_o/\omega)^8] \quad (55)$$

that is, in the expression for the modulus of the fourth-order filter, there are zero coefficients for the second and fourth powers of frequency, with nonzero coefficients for both the eighth *and* sixth powers. This type of response yields a series of alignments, nos. 1–4 of Table I, in which the cutoff frequency (again defined here as the frequency f_3 where the response is 3 dB down) is above the speaker resonant frequency. So also is the box resonant frequency, but again, not to the same extent. As the cutoff frequency is made higher, these alignments require smaller box volumes, and lower values of Q_t.

VI. GENERAL DISCUSSION OF TABLE I

It will be seen that alignments no. 1–9 provide a means of varying the cutoff frequency of a loudspeaker–box combination over a wide range. The last two columns for these alignments illustrate two interesting properties which remain substantially constant ($\pm 5\%$) over this wide range.

1) The expression $C_{as}f_s^2/C_{ab}f_3^2$ is substantially constant around 1.41. This means that if a given speaker for which C_{as} and f_s are constant is placed in different boxes to provide different cutoff frequencies, the box volume will vary with inverse frequency squared. This illustrates a fact long known to designers of vented boxes, but rather blurred by the exponents of "revolutionary new concepts," that the bigger the box, the better the low-frequency response. It is also interesting to note that

$$C_{as}f_s^2 = 1/4\pi^2M_{as} = S_d^2/4\pi^2M_{ms} \cong 1.41C_{ab}f_3^2 \quad (56)$$

that is, for a given cutoff frequency of the combination, the box size varies with the square of diaphragm area S_d^2 and inversely with M_{ms}. In other words, if the mass of the loudspeaker M_{ms} is fixed and the compliance C_{as} is varied to give a different resonant frequency f_s, then the box volume C_{ab} for a given cutoff frequency f_3 remains substantially constant. To this extent, and also in the expression for efficiency (Eq. (66)) the compliance of the loudspeaker is *unimportant.*

2) Q_tf_b/f_s lies around 0.38. If Eq. (18) is rewritten as

$$Q_t = \omega_sM_{as}/R_{at} \quad (57)$$

then the expression above becomes ω_bM_{as}/R_{at} which can be thought of as the total Q of the speaker at the box resonant frequency. This remains nearly constant throughout alignments no. 1–9.

Certain alignments, no. 13, 14, and 27 with no. 12 as a borderline case, which require auxiliary filtering with large attenuation at the cutoff frequency of the whole system, must be considered suspect, since they postulate high acoustic efficiencies in the region of cutoff. Remember that the basis of the theory is that the overall efficiency is low. In the borderline case, no. 12 for example, the peak efficiency will be just above cutoff frequency and will be approximately 2.5^2 times the loudspeaker efficiency. If the loudspeaker is 4% efficient, this means a maximum overall efficiency of 25%. Around this point, the basic assumptions will become inaccurate, especially if resistive losses in the box are large.

Similarly, for reasons of cone excursion (considered in Section X), alignments with smaller values of f_3/f_b such as nos. 17–19 should be avoided if possible. These particular alignments which do give good low-frequency responses in small box volumes would probably be unpopular anyway since they make such great demands on amplifier output in the region of cutoff.

Alignment no. 28 is interesting in that it represents the result of "pure" bass lift. In the other alignments which use "amplifier aiding," the response often rises near cutoff, but always falls off ultimately at lower frequencies at a rate of 6 or 12 dB per octave. In this way, although increased amplifier output may be required over a comparatively narrow range of frequencies, a greatly decreased output, and with it, a greatly decreased cone excursion, is required at the lower frequencies. But in alignment no. 28, a simple low-frequency lift of 6 dB, such as results from a network with two resistors and a capacitor, is required. The mean frequency of lift (at which the lift is 3 dB) is $1.08f_3$. However, since the maximum lift continues to the lowest frequencies, the amplifier would be more likely to cause intermodulation distortion with "rumble" components. However it does give some decrease of box volume compared with alignment no. 5.

It should be emphasized that these alignments are by no means the only ones possible. They have been chosen as the ones most likely to be useful and as showing the

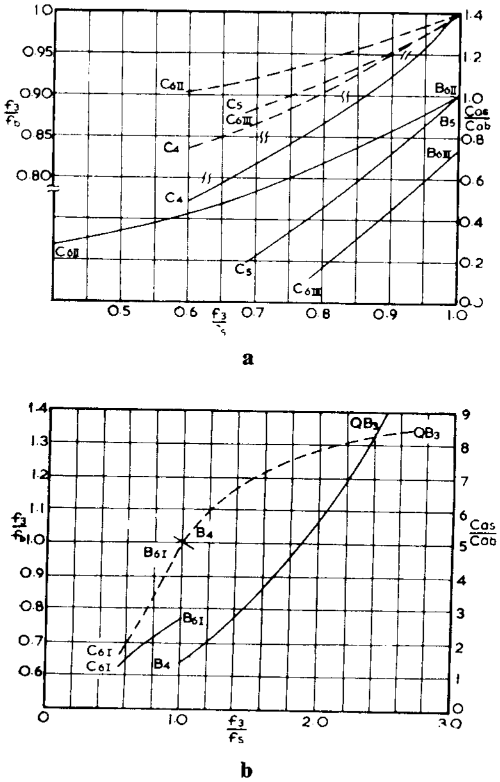

Fig. 7. f_3/f_b (dashed curves) and C_{as}/C_{ab} (solid curves) versus f_3/f_s. **a.** For design of medium and large boxes; alignment types B_4–C_4, B_5–C_5, and B_6–C_6 class II and III. **b.** For design of small boxes; alignment types QB_3–B_4 and B_6–C_6 class I.

trend of results. If more sophisticated filtering in the amplifier is possible, the choice widens greatly, e.g., there are six alignments for the eighth-order Butterworth response, each with its fourth-order amplifier filter and the ratios C_{as}/C_{ab} of 0.518, 0.681, 1.000, 1.316, 1.932, and 2.543.

Another possibility would be the use, instead of the "quasi-Butterworth" responses, of "sub-Chebyshev" responses, i.e., response functions derived by multiplying the real coordinates of the Butterworth poles by a constant k which is *greater* than 1.

In answer to the question proposed in 1) of Section I—What is a large box?—it would appear that a medium sized box would be one for which V_b is about the same value as V_{as}, say C_{as}/C_{ab} lies between 1 and 1.414. For large boxes, C_{as}/C_{ab} is less than 1, for small boxes C_{as}/C_{ab} is greater than 1.414. Table I shows that smaller boxes demand a smaller value of Q_t. Thus if Q_t is not properly controlled, the smaller boxes will tend to cause a greater peak at f_h, while larger boxes will cause the peak to diminish. Fig. 7 is plotted from the points of Table I. Typical response curves for alignments no. 3, 5, and 8 are given in Fig. 8.

VII. TO DESIGN A BOX FOR A GIVEN LOUDSPEAKER

First, the following three loudspeaker parameters must be known: 1) the resonant frequency f_s, 2) the Q values Q_a and Q_e, the latter being usually the controlling factor. This is discussed in more detail in Section IX, Eqs. (71) and (72), and 3) the acoustic compliance C_{as}. This is expressed most conveniently as V_{as}, the volume of air whose acoustic compliance is equal to that of the speaker.

Since in general the acoustic compliance, from [3, Eq. (5.38)] is given by

$$C = V/\rho_0 c^2 \tag{58}$$

then

$$C_{as}/C_{ab} = V_{as}/V_b \tag{59}$$

where V_b is the volume of the box.

The design is commenced in one of two ways:

1) If the box size is limited, V_b is taken as the assigned value. Remember this is the net volume, and that the bracing and the volume displaced by the loudspeaker and the vent (say 10%) must be subtracted from the gross volume. From this value and the known value of V_{as}, the ratio C_{as}/C_{ab} is found, and thence either from Fig. 7 or interpolation from Table I, the values of f_3/f_s, f_3/f_b, and Q_t. Hence f_3 and f_b are found.

2) If a certain frequency response is required, then f_3 is the starting point. The ratio f_3/f_s is found, then from Fig. 7, or by interpolation from Table I, f_3/f_b, C_{as}/C_{ab}, and Q_t. Hence f_b and V_b are found.

The choice of alignment will depend largely on what can be done with the amplifier circuits. For a straightforward amplifier with no filtering, alignments no. 1–9 would be chosen. If a slightly larger box is possible, alignments no. 10 and 11, with their simple CR input filtering make it possible to ease the power handling requirements of both speaker and amplifier. If a more sophisticated design of input filtering is possible as described in Sections V and XII, alignments 15–17 can be used to obtain good acoustic output from small boxes at the expense of higher electrical power output from the amplifier, while alignments no. 20–25 are the most suitable if a fair sized box is available and only moderate lift is required from the amplifier, although in all the fifth- and sixth-order cases, the power required from the amplifier and the excursion demanded of the speaker decrease rapidly below cutoff.

Having found f_b and V_b, the vent dimensions may be found using the methods of the standard texts [8]. However, the following adaptation of the method has proven useful for calculation. The standard form is

$$V_b = 1.84 \times 10^8 S_v/\omega_b{}^2 L_v \tag{60}$$

where S_v is the cross-sectional area of the vent, in square inches, and L_v is the effective length of the vent, in inches, which includes its actual length together with an end correction.

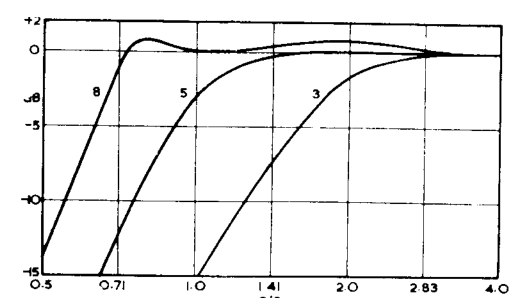

Fig. 8. Typical response curves for identical loudspeakers, but different box sizes. C_{as}/C_{ab} = 0.56, 1.41, and 4.46, corresponding to alignments no. 8, 5, and 3 (types C_4, B_4, and QB_3) of Table I.

This is written more conveniently as

$$L_v/S_v = 1.84 \times 10^8/\omega_b{}^2 V_b. \qquad (61)$$

The quantity L_v/S_v, which has the dimension of inches^{-1}, is equivalent to an inductance (acoustic mass) which resonates at ω_b with a capacitance (acoustic compliance) equivalent to V_b. When L_v/S_v is found, a value is chosen for the vent area S_v. It has been shown already in connection with Eq. (6) that the radiation resistance, and therefore the operation of the vented box, is independent of the value of S_v. Now it is usually stated that S_v should normally be the same as the effective radiating area of the cone [8], i.e., S_d. However, this will often involve an excessive length of vent, especially in small boxes and at low cutoff frequencies, because, since L_v/S_v is fixed, the volume L_vS_v displaced by the vent varies as $S_v{}^2$. At the same time, a small amount of distortion is generated in the vent (see [4, Eq. 6.33]) which is a maximum near the box resonant frequency ω_b and is proportional to L_v. On the other hand, Novak [2] quotes 4 in^2 as the lower unit.[2] As shown before, a small area vent has still a high value of Q. However, it will also have higher alternating velocities of air, and this will limit the amount of acoustic power that can be handled linearly. The only advice that can be given is to design the vent area as large as possible in the particular circumstances, up to a limit equal to the piston area.

The maximum length of L_v is usually quoted as $\lambda/12$ where λ is the wavelength of sound at the loudspeaker resonant frequency f_s. The actual requirement is that the vent, which is essentially a transmission line, should look like a lumped constant mass at all the frequencies for which the box is effective. That is, it must still be rather shorter than $\lambda/4$ at frequencies somewhat above f_h of Fig. 5. The value of f_h with respect to f_s will depend on the box tuning. But it also varies with C_{as}/C_{ab}; with a smaller box, f_h is higher.

With the chosen area of vent, first calculate the part of L_v/S_v due to the end correction. This length L'' is usually quoted as

$$L'' = 1.70R \qquad (62)$$

where R is the effective radius of the vent, i.e.,

$$(L_v/S_v)_{end} = 0.958/\sqrt{S_v}. \qquad (63)$$

This applies to pipes with both ends flanged. When a free-standing pipe is used, the end correction is

$$L'' = 1.46R \qquad (64)$$

and

$$(L_v/S_v)_{end} = 0.823/\sqrt{S_v}. \qquad (65)$$

In a pipe the end correction is not usually a large part of L_v/S_v. It forms the larger part when the vent is a simple hole in the front panel and then Eq. (63) is correct.

A method favored by the writer, if styling permits, is to build a shelf into the bottom of the box as in Fig. 9, with a spacing l from the back panel equal to the height

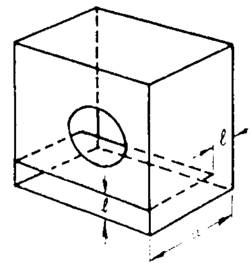

Fig. 9. Simple method of making a tunnel or duct.

of the opening in the front panel. In this case, the effective length of the tunnel is the box depth d plus the end correction as given by Eq. (62) and allowances for thickness of lumber. This vent is tuned by varying l.

When $(L_v/S_v)_{end}$ is found, it is subtracted from the required value of L_v/S_v, and from this, the actual length L_v' is calculated. If this value is unsuitable, another value of S_v is tried and so on (see Appendix).

With regard to box dimensions, it is desirable to take all precautions to prevent strong standing waves. If a corner box is made, the problem is usually fairly easy to solve since the box sides are splayed at least in two dimensions. If a rectangular box is made, and if styling allows, the inside dimensions should be in the preferred ratio for small rooms, that is, 0.8:1.0:1.25 or 0.6:1.0:1.6. In any case, the speaker should be mounted away from the center of the front panel.

The need for sound sealing, with good glued joints, adequate bracing, and adequate damping of the internal surfaces has been stressed often before, so no more need be said of it here. The same is true for the improvement in performance that is obtained by placing the box in the corner of the room, and also by building the sides of the box right down to the floor. However, this last does not seem to be realized sufficiently and the current fad for mounting all furniture on legs causes much unnecessary loss of performance in loudspeaker boxes.

Finally the value of Q_t required by the alignment is compared with the values Q_a and Q_e available, and suitable adjustments are made to the amplifier to achieve a correct overall Q_t. This is dealt with in Section XII, and a worked example is given in the Appendix.

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[2] This is presumably for the particular case he considers where f_b is 25 Hz, and the acoustic output power is high. For a higher box resonant frequency and/or lower power, an even smaller vent area seems permissible.

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Neville Thiele was educated at the University of Queensland and the University of Sydney, graduating as Bachelor of Engineering in 1952. He joined the staff of E.M.I. Australia Ltd. in 1952 as a development engineer in the Special Products Division. During 1955 he spent six months in England, Europe and the United States and on return was responsible for the development of E.M.I. Australia's television receiver. He was appointed advance development engineer in 1957.

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Loudspeakers in Vented Boxes: Part II*

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Australian Broadcasting Commission, Sydney, N.S.W. 2001, Australia

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VIII. LOUDSPEAKER EFFICIENCY

In Eq. (12) an expression was derived for the efficiency of a loudspeaker in a box, which consists of three parts. We have considered, in the meantime, the third part which varies with frequency. We now consider the first two parts. Thus the basic efficiency

$$\eta_{ob} = (\rho_o/4\pi c)(B^2 l^2 S_d{}^2/R_e M^2{}_{ms}). \tag{66}$$

If this experience is compared with Beranek's Eq. (7.19) it will be seen to give one quarter of his value, after the differences in notation are allowed for.

1) Multiplication by 100 to give percentage.

2) The definition of "nominal input power" in Eq. (10) of this paper as the power delivered by the amplifier into the nominal speaker impedance R_e[3]. Beranek's treatment is based on the idea of maximum power transfer when the load impedance is equal to the generator impedance, as in his Eq. (7.14). If this condition, $R_g = R_e$, is substituted in his Eq. (7.19), one of the conditions for agreement with Eq. (66) is satisfied. However, in dealing with the output power from an amplifier, the writer prefers to consider the power delivered into the load without regard to the output impedance R_g, for the relationship of R_g to the optimum load impedance depends in the first place on the nature of the output device, transistor, pentode, or triode. Furthermore, R_g can be manipulated by feedback techniques (see Section XII) to almost any desired value without affecting the condition for optimum output power. Hence the treatment in this paper.

3) The lumping in this paper of all mechanical mass into M_{ms}.

The additional multiplication factor of one quarter arises from the following.

4) Beranek's figure being for the radiation from *both* sides of the diaphragm, giving twice the output from one side.

5) The assumption in this paper that the radiation resistance in a box is that of a piston at the end of a long tube [3, p. 216]. This radiation resistance is one half of that of a piston in an infinite baffle.

Thus the results are consistent. We will continue here to use η_{ob}, unless stated otherwise. But it is important to define efficiency in terms of actual use and to remember that the value of η_{ob}, being the basic efficiency in a box, is one half the efficiency on an infinite baffle and one quarter of the efficiency, if radiation from both the front and back of a speaker in an infinite baffle is considered.

To simplify the understanding of Eq. (66), we make a further substitution. It can be shown that

$$l^2/R_e = V_{cu}/2\sigma \tag{67}$$

where σ is the resistivity of the conductor and V_{cu} is the volume of the conductor assumed to be completely within the air gap. In so far as the conductor overlaps the air gap a correction factor would be applied. Then

[3] The nominal impedance of a loudspeaker is usually taken as the minimum impedance at mid-frequencies, at f_n in Fig. 5. This is a little greater than R_e; but for simplicity, and it is hoped without too much confusion, the nominal impedance is taken here as R_e.

* Reprinted from *Proceedings of the IRE Australia*, vol. 22, pp. 487–508 (Aug. 1961). For Part I see *J. Audio Eng. Soc.*, vol. 19, pp. 382-392 (May 1971).

Eq. (66) becomes

$$\eta_{ob} = (\rho_o/8\pi c\sigma)(B^2 S_d^2 V_{cu}/M^2_{ms}) \tag{68}$$

that is, once the voice coil conductor material, and therefore σ, is chosen, the loudspeaker efficiency depends on the four parameters in the second bracket. Without digressing too far into the problem of loudspeaker design, it is noted that this shows the two basic questions in loudspeaker design for good efficiency at low frequencies.

1) How to make the product B^2V_{cu} a maximum for a given magnet, since the larger V_{cu} is made, the wider and/or deeper is the air gap, and hence the lower is B.

2) How to make S_d^2/M_{ms}^2 a maximum, since the larger the area the greater the mass for a given cone thickness. If thickness is reduced, break-up problems increase due to nonlinearity of the piston drive. In conventional designs the mass of the voice coil is small (less than 20%) compared with the mass of the cone, so there is little interaction between V_{cu} and M_{ms}.

The writer prefers to express efficiency as an electroacoustic conversion loss

$$\mathrm{dB}_{ea} = 10 \log_{10} \eta. \tag{69}$$

For example, 1% efficiency is equivalent to 20-dB electroacoustic conversion loss. This facilitates comparisons between different designs and estimations of the acoustic level (in phons) which a speaker will provide with a given amplifier and listening room (see Appendix).

IX. RELATIONSHIP OF EFFICIENCY η, Q, AND BOX VOLUME

First we take Eq. (57) and break Q_t into two component parts, one due to the acoustic resistances and the other due to electrical damping, so that

$$1/Q_t = 1/Q_a + (1/Q_e)[R_e/(R_g+R_e)]. \tag{70}$$

Then from Eqs. (8) and (57), the acoustic Q of the loudspeaker

$$Q_a = \omega_s M_{as}/R_{as} \tag{71}$$

and the electrical Q of the loudspeaker

$$Q_e = \omega_s M_{as} R_e S_d^2/B^2l^2 \tag{72}$$

i.e.,

$$Q_e = 2\sigma\omega_s M_{ms}/B^2V_{cu}. \tag{73}$$

Again if we consider the approximate relationship established in Table I that

$$C_{as}f_s^2/C_{ab}f_3^2 \cong \sqrt{2} \tag{74}$$

thus, converting the acoustic compliance of the box into the equivalent volume of air, the box volume

$$V_b \cong (\rho_o c^2/\omega_3^2\sqrt{2})(S_d^2/M_{ms}) \tag{75}$$

remembering that this approximate relationship holds only in the absence of amplifier assistance.

Now considering together Eqs. (68), (73), and (75), the following points emerge.

1) The same considerations that ensure high efficiency also ensure a low Q_e, except that Q_e is independent of the projected piston area S_d and depends only on the *first* power of the cone mass M_{ms} instead of the second power.

2) The box volume depends, apart from the choice of cutoff frequency f_3, only on S_d^2 and M_{ms}. Reduction of box volume by reduction of S_d involves an increased cone excursion, which is inversely proportional to S_d and ω_b^2, for a given acoustic power. If the box volume is reduced by increasing M_{ms}, η is decreased even more (see Eq. (68)), necessitating increased amplifier power. It would seem that the well-known R–J enclosure works this way. The opening in front of the cone is restricted, and this increases the air mass loading M_{a1} of Fig. 1 in the same manner as a vent. Thus M_{ms} is increased and the box volume V_b, i.e., C_{ab}, for a given low-frequency cutoff is reduced, but at the price of reduced efficiency throughout the piston range.

3) The best way of increasing η and lowering Q_e is to increase the flux density B. But if one starts with a reasonably high value of B in the first place, the cost of obtaining an extra decibel of efficiency increases rapidly. So again to obtain a given amount of acoustic power at a given price, a compromise must be struck between the sizes of magnet, box, and amplifier. However, this discussion does show the reason for the large magnet, long throw, heavy cone designs used overseas in small "bookshelf boxes."

Note that Q_a in Eq. (71) depends only on acoustic reactance and resistance, that is, Q_a is independent of B.

Substituting Eqs. (58) and (73) in (68), we obtain the interesting relationship

$$\eta_{ob} = \omega_s^3 V_{as}/4\pi c^3 Q_e \tag{76}$$

where V_{as} is the volume of air equivalent to the acoustic compliance of the loudspeaker, or

$$\eta_{ob} = 8.0\times 10^{-12} f_s^3 V_{as}/Q_e \tag{77}$$

where V_{as} is in cubic inches. *Thus the basic efficiency of the speaker can be calculated from the three parameters which are used for the design of the box.* A physical explanation of the variation of η and Q_e is given at the end of Section XII.

X. EXCURSION OF LOUDSPEAKER CONE

In the derivation of Eq. (12) it was found that

$$\begin{aligned} U_c/(U_c-U_p) &= 1-1/\omega^2 M_{ar}C_{ab} \\ &= 1-(\omega_b/\omega)^2. \end{aligned} \tag{78}$$

Thus the acoustic output power radiated by the cone alone is

$$W_{aoc} = W_{ei}\eta_{ob}[1-(\omega_b/\omega)^2]^2|E(j\omega)|^2. \tag{79}$$

Now starting from the relationship

$$W_{aoc} = (R_{ma}\dot{x}^2)10^{-7} \tag{80}$$

which is [4, Eq. 6.13], where R_{ma} is the mechanical radiation resistance and $\dot{x}$ is the rms velocity of the piston in cm/s, it is possible to derive an expression for peak cone movement,

$$x_{pk} = 1.31\times 10^5\sqrt{W_{aoc}}/f^2S_d \tag{81}$$

or

$$x_{pk} = 5.17\times 10^6\sqrt{W_{aoc}}/\omega^2S_d \tag{82}$$

where x_{pk} is in inches (note that this x which stands for excursion is unrelated to the shape parameter x of Eq.

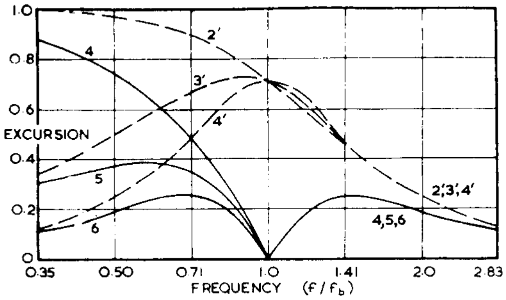


Fig. 10. Normalized cone excursion versus normalized frequency for various orders of Butterworth response with loudspeaker in vented box (solid curves) and in infinite Baffle (dashed curves). Curves are numbered for order of response. Normalized excursion is $|(f_b/f)^2 - (f_b/f)^4| \cdot |E(j\omega)|$, part of Eq. (84).

(21) *et seq.*), S_d is in square inches, and W_{aoc} is in watts. Again allowance is made for the fact that the loudspeaker is mounted in a box so that the radiation resistance is half the value for an infinite baffle. Thus Eqs. (81) and (82) will give values for displacements which are $\sqrt{2}$ times those given in [4, Fig. 6.9]. Thus

$$x_{pk} = 5.17\times 10^{6}(\eta_{ob}W_{ei})^{1/2}[1-(\omega_b/\omega)^2]|E(j\omega)|/\omega^2 S_d. \quad (83)$$

If we write this expression as

$$x_{pk} = [1.31\times 10^{5}(\eta_{ob}W_{ei})^{1/2}/f_b^2 S_d][(\omega_b/\omega)^2-(\omega_b/\omega)^4] |E(j\omega)| \quad (84)$$

it is apparent that there are two parts, one fixed for a given speaker and box (note frequency f_b in this expression) and one that varies with frequency. This latter expression is plotted in Fig. 10 for various Butterworth responses, in which box, speaker, and cutoff frequencies are identical. The solid curve 4 gives the excursion of the classical fourth-order Butterworth alignment no. 5 of Table I. Solid curve 5 refers to the fifth-order Butterworth alignment no. 10, which includes a simple auxiliary filter. Solid curve 6 refers to the sixth-order Butterworth alignment which is identical for nos. 15, 20, and 26, since both frequency response and box resonant frequency are the same in each. For comparison, the dotted curves give the excursions for the same speaker in an infinite baffle (totally enclosed box) with the same power. Dotted curve 2 applies to a speaker with a second-order Butterworth response ($Q_t = 0.707$). Dotted curve 3 applies to a third-order Butterworth response ($Q_t = 1$, with a simple auxiliary filter). Dotted curve 4 applies to a fourth-order Butterworth response ($Q_t = 1.307$, with a second-order auxiliary filter). The frequency response is the same as solid curve 4, but it is obtained by different means. The curves show the following.

1) The excursion below resonance is reduced greatly in both vented box and infinite baffle when an auxiliary highpass filter is used. The first-order auxiliary filter gives a good improvement especially in view of its simplicity. The second-order auxiliary filter not only allows a greater reduction of cone excursion, it also allows the use of three separate box alignments for the same response and allows box volume to be traded for amplifier power in the case of the vented box. The Butterworth curves with second-order auxiliary filters are symmetrical about the center frequency. There seems little need therefore to use more elaborate filtering.

2) Even more important, the excursion of the cone is reduced greatly when the loudspeaker is placed in a vented box. The curve predicts zero excursion at the box frequency. This arises from the assumption that the Q of the box circuit is infinite. While this cannot be achieved completely in practice, the excursion at the box frequency will be low so long as the ratio of Q of the box to Q of the speaker is high, as demonstrated in Section II.

Of course, if resistance is deliberately introduced into the box circuit, as by making the vent from a number of small holes or by stretching fabric across the vent, the Q will be greatly reduced and some of the advantage of the vented box will be lost, as shown in the next section. Fig. 10 refers only to Butterworth responses. In Fig. 11, a plot is made of the function $|(\omega_b/\omega)^2-(\omega_b/\omega)^4|$ against frequency. If, for example, in a Chebyshev response the frequency response is known, the excursion at different frequencies can be found by reading off the function at a given frequency on Fig. 11 and multiplying it with the frequency response. The rapid rise of the function between normalized frequencies of 1 and 0.71 shows why responses should be preferred in which f_b is not too much greater than f_3. Thus with respect to cone excursion, an alignment in the group 20–25 would be preferred to its counterpart in the group 15–19 which has a lower value of f_3/f_b.

It would seem that in published ratings of loudspeakers, the maximum excursion x_{max} would be more useful than the conventional rating of maximum input power. The latter might save the loudspeaker from a melted voice coil, but when mechanical damage or undistorted acoustic output are of interest, x_{max}, along with the kind of baffle and the alignment, determine the performance.

XI. BOXES WITH RESISTIVE LOADING OF VENT

Good results have been reported with resistively loaded vents [1]. These were therefore investigated using both series and parallel loading of the vent as shown in Fig. 12. In both cases, the resistance was assumed to be constant with respect to frequency and the response function was found to be of third order.

This, by the way, explains a discrepancy between the statements in [3, p. 244] and in [2, p. 11] that the drop in response below cutoff is 18 dB per octave, even though [2, Eq. 15)], which is equivalent to Eq. (20) of this

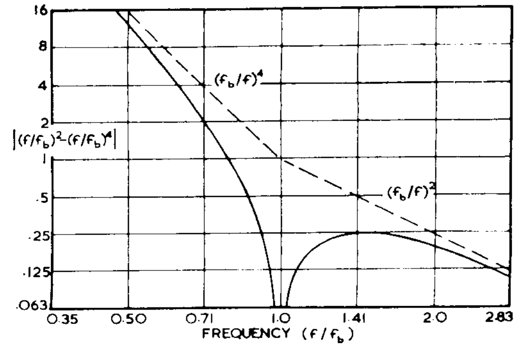


Fig. 11. Function $|(f_b/f)^2 - (f_b/f)^4|$ versus normalized frequency f/f_b. The function, part of Eq. (84), is used to compute excursion when frequency response $|E(j\omega)|$ is known.

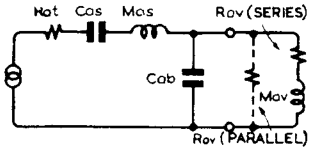


Fig. 12. Equivalent acoustic circuit of loudspeaker and box showing added acoustic damping in series or parallel with vent.

paper, obviously has an asymptotic slope of 24 dB per octave. In the practical case, where resistance loading of the vent however small will be encountered, the asymptotic slope will eventually be 18 dB per octave; but so long as the original simplifying assumptions hold, the response in the region that concerns us will be effectively 24 dB per octave.

The expressions are, for the case of series resistance loading,

$$E(p) = 1/\{1+(1/p)(1/Q_bT_b+Q_bT_b/T^2_s)+(1/p^2)(1/T_s^2+1/T_b^2+C_{as}/C_{ab}T_s^2)+Q_b/p^3T_s^2T_b\} \quad (85)$$

when

$$1/Q_t = T_s/Q_bT_b+Q_bT_b/T_s \quad (86)$$

and Q_b is defined as the ratio of acoustic mass resistance to series acoustic resistance of the vent at the *box* resonant frequency.

For the case of parallel resistance loading,

$$E(p) = 1/\{1+(1/p)(Q_bT_b/T_s^2+1/Q_bT_b+C_{as}T_b/C_{ab}T_s^2Q_b)+(1/p^2)(1/T_s^2+1/T_b^2+C_{as}/C_{ab}T_s^2)+Q_b/p^3T_s^2T_b\} \quad (87)$$

when

$$1/Q_t = Q_bT_b/T_s+T_s/Q_bT_b+C_{as}T_b/C_{ab}T_sQ_b \quad (88)$$

and Q_b in this case is the ratio of parallel acoustic resistance across the vent (series resistance being assumed negligible) to acoustic mass reactance. Note the inversion of the expression for parallel Q_b compared with that for series Q_b. Since these equations are of third order and there is one extra variable Q_b, there are two extra degrees of freedom in the design. However, one is removed if an all-pole function is desired, hence Eqs. (86) and (88). Before an alignment is commenced, one other parameter must be fixed arbitrarily. The ratio C_{as}/C_{ab} seems the easiest to handle for this purpose. Thus in a third-order Butterworth alignment, if C_{as}/C_{ab} is made 1.414, for comparison with the fourth-order Butterworth alignment no. 5 of Table I, the results are as given in Table II.

Table II. Parameters for third-order Butterworth alignment with resistive vented loading.

Method of Loading	f_3/f_s	f_3/f_b	C_{as}/C_{ab}	Q_t	Q_b
Series Resistance	1.317	1.285	1.414	0.379	2.22
Parallel Resistance	1.420	1.120	1.414	0.352	2.25
No Resistance (Alignment No. 5, for comparison)	1.000	1.000	1.414	0.383	∞

It will be seen that although the box had the same volume, the cutoff frequencies for the resistively loaded alignments are 1.32 and 1.42 times higher than no. 5 of Table I. Compared with previous alignments (no. 1–9 of Table I) those of Table II are most inefficient in utilization of box volume, there is no compensating freedom to use a larger value of Q_t, in fact it needs to be a little smaller, finally and more important, the excursion of the speaker near cutoff frequency is greatly increased. For these reasons, the use of acoustic damping seems to be unjustified. It is realized that the cases treated here use resistances which are constant with frequency. Some acoustic resistances, as described for example in [3, Eqs. (5.54) and (5.56)], vary with frequency and might have a somewhat different effect. However, the use of added damping with the attendant dissipation of input power seems to be wrong in principle, unless a suitable alternative cannot be found. It is believed that the method outlined already provides the suitable alternative.

Effect of Losses in Box and Vent

Having established that intentional loading of the vent is undesirable, it is of interest to know the effect on the ideal response, obtained by assuming zero loss, of small unavoidable losses in the box and vent. We will only consider performance at the box resonant frequency, since at this frequency 1) the box circuit contributes most, in the ideal case all, of the acoustic output, and 2) the losses in the box circuit are greatest.

In the ideal case, the transfer impedance connecting the input force $E_gBl/S_d(R_g+R_e)$ with the vent volume velocity U_p in Fig. 2, at the box resonant frequency ω_b, is $j\omega_bM_{av}$. If now we express all the losses in the vent *and* the box as Q_b, the "Q of the box and vent circuit," the transfer impedance, and thus the frequency response at ω_b is reduced by a factor which we will call the maximum box loss $(A_b)_{max}$. Then, to a close approximation,

$$(A_b)_{max} = 1/[1+(1/Q_tQ_b)(C_{ab}/C_{as})(\omega_b/\omega_s)]. \quad (89)$$

If we apply the approximations of parts 1) and 2) of Section VI for the "unassisted" alignments no. 1–9 of Table I, Eq. (89) is simplified to

$$(A_b)_{max} = 1/[1+(1.85/Q_b)(f_b^2/f_3^2)] \quad (90)$$

that is, for a given value of Q_b, the box loss increases with higher values of f_b/f_3 and thus, larger box sizes.

To illustrate the effect of box loss, Eq. (89) is applied to various alignments. Taking first the classical alignment, no. 5 of Table I, the maximum box loss is 0.5 dB when Q_b is 30 and 1.5 dB when Q_b is 10. Taking other, extreme, alignments when Q_b is 30, the losses for alignments no. 1, 9, 19, and 25 are 0.3 dB, 0.7 dB, 0.5 dB, and 0.7 dB, respectively. Thus it can be seen that a Q_b of 30 will have little effect on any alignment. With a Q_b of 10, the losses are 0.9 dB, 1.9 dB, 1.5 dB, and 2.2 dB, respectively, i.e., when the box Q is reduced three times, the maximum box loss is increased approximately three times in each case. A method of measuring Q_b is given at the end of Section XIV and illustrated in the Appendix.

Table III. Change of output impedance R_g with type of feedback.

	Negative	Positive
Voltage Feedback	R_g Decreases	R_g Increases
Current Feedback	R_g Increases	R_g Decreases

XII. AMPLIFIER CIRCUITS

Negative Output Impedance

It is essential to the method that the overall Q_t of the loudspeaker plus amplifier be properly controlled within $\pm 10\%$ for ± 1 dB accuracy of response. As explained in Section V, if Q_t is twice the optimum value, a 6-dB peak results. Similarly if Q_t is too small, there will be a dip in the response. Thus it is important that the speaker Q_e be known, either from information supplied by the manufacturer or by measurement, and that the amplifier output impedance be then adjusted to give the required overall value of Q_t. It is assumed in the following that the available speaker Q_e is larger than the required Q_t. This is the more usual case, especially with lower priced loudspeakers. But if it is smaller, a suitable adjustment can easily be made, for example, by changing the positive current feedback to negative current feedback.

The subject of amplifier output impedance control properly requires another paper, which it is hoped will be presented later. For the present only some general results will be given.

If feedback is applied to an amplifier, not only does its gain change, but its effective output impedance R_g changes also; not its optimum load impedance which remains unchanged by feedback but the impedance which is seen when looking back into the amplifier output terminals. The effect of applying different kinds of feedback is shown in Table III.

The terms voltage feedback and current feedback refer of course to feedback of a voltage which is proportional to output voltage and output current, respectively. In the latter case, this is usually achieved by placing a small resistor in series with the load, and taking the voltage drop across it for feedback. It will be seen that not only does negative voltage feedback reduce the output impedance R_g, positive current feedback reduces R_g also, and to the greater extent that R_g can be made zero or negative.

Negative output impedance is characteristic of oscillators; one therefore tends to be wary of it as tending to instability. But this can only happen when the positive output impedance presented by the load is less than the negative impedance presented by the amplifier. Now the impedance of a loudspeaker in a box, typified by Fig. 5, can never be less than its dc resistance R_e of Fig. 4. The only exception is at very high frequencies, where the shunt capacitance of the connecting leads takes effect. But unless the leads are very long and the nominal impedance of the speaker is high, this will not usually take effect within the bandwidth of the amplifier. And in any case, we will want to eliminate the negative impedance characteristic at the higher audio frequencies for reasons that will be discussed later. Thus a negative impedance amplifier can be made completely stable apart from gross misadjustment, such as connecting a loudspeaker of much lower impedance than the design figure or short-circuiting the output leads.

The method of applying mixed feedback is shown in Fig. 13. It will be seen that if the sense of the voltage developed across the potential divider R_3 and R_4 is negative, then the voltage developed across the current feedback resistor R_2, usually made less than 1/10 the nominal impedance of the speaker to minimize power loss, will be positive. The circuit shows why this method is sometimes described as bridge feedback. Usually the circuit is arranged to be unbalanced at all frequencies so that the net feedback is always negative, but it need not necessarily be so. For example, if no net negative feedback is desired, so that there is no overall gain reduction with nominal load, the bridge will be balanced at nominal load.

Physically, the circuit can be thought of as having a certain amount of feedback with nominal load, in which the negative voltage feedback is partially neutralized by the voltage from the positive current feedback resistor. If the impedance Z_1 is open-circuited, the current feedback from R_2 disappears leaving a greater amount of negative feedback. Thus the output voltage may be less on open circuit than on nominal load. This is the effect we describe as negative output impedance. Its extent, or whether it is seen at all, will depend on the original gain and output impedance of the amplifier and the value of the feedback resistor R_2. Thus if we have, as in Fig. 4, a loudspeaker resistance R_e, and make the effective output impedance of the amplifier R_g equal to, say, $-0.6R_e$, the total effective impedance of R_g+R_e becomes $+0.4R_e$. And if the Q_e of the loudspeaker is 1.0, this will make the overall Q_t a value of 0.4 by applying a maximum of 1.0/0.4 times, i.e., 8.0 dB, extra gain reduction by negative feedback when the impedance of the speaker becomes high, as at f_h and f_l of Fig. 5. (Need it be emphasized that this form of damping does not dissipate amplifier output power, except in the small current feedback resistor. It reduces power by feedback at the source.)

This fact necessitates a degree of additional care in the design of negative impedance amplifier. For when the load is open-circuited, the negative feedback rises to the maximum; in this case a gain reduction of 8 dB above the nominal value, and the stability margin will be reduced. The size of the negative impedance will in practice be limited either by this consideration or by the need for a feedback resistor so large that it dissipates an appreciable part of the output power.

An alternative method of control damping uses a feedback winding closely coupled to the voice coil. In this way, feedback can be taken effectively from the junction of R_e and L_e in Fig. 4. Simple negative feedback

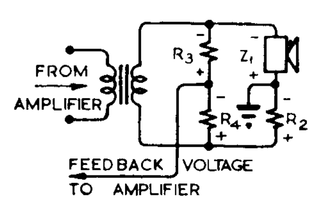
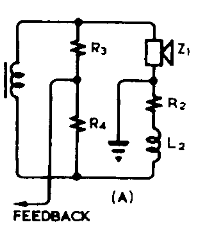


Fig. 13. Method of applying mixed feedback (positive current and negative voltage).

then reduces an effective output impedance which is the sum of $R_g + R_e$. Thus Q_t is reduced in the same way as before. Since the impedance of the feedback circuit is usually high compared with the voice coil impedance, the feedback winding can be made of very fine wire. In fact, if it is wound bifilar with the main winding with wire 16 B&S gauges smaller, it will fit into the air spaces between the larger wires. It thus takes up no more space in the air gap and adds less than 3% to the mass of the copper in the voice coil. Unfortunately, such a winding is difficult to achieve in production and is thus rarely, if ever, used.

If negative impedance is applied, it reduces the output voltage whenever the load impedance is high, i.e., not only in the region of f_l and f_h in Fig. 5, but also at frequencies above f_n where the impedance rise is due to the inductance L_e of Fig. 4. At high frequencies, this contributes nothing to the acoustic damping of the speaker, but simply reduces the high-frequency response, in the case quoted above, a maximum of 8 dB. This is usually undesirable, so the negative impedance should be eliminated at the higher audio frequencies. One method among several possible is shown in Fig. 14**a**. Here an inductance L_2 is added to the feedback resistor R_2 with a time constant L_2/R_2 matching that of the speaker, usually in the range of 30–60 μs. This can be easily done by winding a solenoid of copper wire which combines resistance R_2 and inductance L_2. However, since this achieves its result by feeding back an increasing positive voltage to neutralize an increasing negative voltage, quite small unbalance between the two can cause instability at high frequencies.

On the other hand, consider the circuit of Fig. 14**b** where the lower resistor of the negative feedback potential divider R_4 becomes two resistors R_5 and R_6 in series. Suppose that a suitable set of resistors R_2, R_3, and R_4 has been found to give the correct gain and output impedance for low frequencies with the dotted connection open-circuited. It is then possible to find a tapping point on R_4 (i.e., the junction of R_5 and R_6) such that the same gain is obtained on nominal load whether the dotted connection is open circuit or short circuit. This is done by connecting the nominal load and making R_5 and R_6 a potentiometer whose wiper is grounded through a switch. The wiper is adjusted until the gain is the same with the switch open or closed. In the open-circuit condition, the output impedance will be the value originally chosen, but on short circuit, most of the positive current feedback will be eliminated. If then a capacitor is substituted for the switch as shown in Fig. 14**b**, the output impedance will change from a negative value at low frequencies to a small value, either positive or negative depending on the particular circuit. The frequency of changeover, which should be, say, two octaves above f_h, depends on the capacitance C and the resistances R_5 and R_6. At the same time, the gain of the amplifier on nominal load stays constant over the whole audio range.



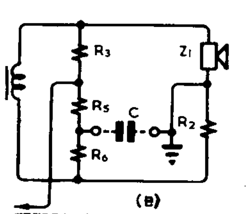


Fig. 14. Methods of eliminating negative output impedance at high frequencies.

Auxiliary Filters

The auxiliary filtering needed for sixth-order alignments is best provided by circuits using *RC* networks in a feedback loop ahead of the main amplifier. In general it is unwise to use the main amplifier feedback loop to provide both negative impedance and high-pass filtering. It is hoped to deal with this in a later paper, but for the moment the reader's attention is directed to the extensive literature, of which [9] and [10] are examples, concerning low-frequency filters without inductors, which use resistors, capacitors, and tubes in comparatively inexpensive combinations.

Maximum Power at Maximum Impedance

The electrical impedance seen at the terminals of a loudspeaker varies greatly with frequency, but output stages deliver maximum power into a comparatively narrow range of impedances. To consider the maximum acoustic power that can be delivered by an amplifier through a loudspeaker, we return to the equivalent electrical circuit of Fig. 4, together with the impedance curve of Fig. 5. For this purpose, we ignore for the moment the inductance L_e with its electrical shunt loss R_{sh} and assume that the curve of Fig. 5 reaches a final value of R_e above f_n.

The acoustic output depends on the voltage across R_{es}, which includes the electrical equivalent of the radiation resistance R_{ar1}. Since R_{ar1} varies with frequency squared, the voltage across R_{es} needs to vary inversely with frequency to maintain constant acoustic power. At the higher frequencies the motional impedance is much lower than R_e and is controlled by the reactance of C_{mes}, which is equal to B^2l^2/M_{ms}. Thus the condition for flat response is achieved, often described as mass control.[4]

If B is varied while R_e remains constant, the motional impedance at any given high frequency within the piston range will increase with B^2. The electrical equivalent of radiation resistance, though small, will increase and with it the ratio, again small, of acoustic power radiated to electrical power input. Thus efficiency varies with B^2. At the same time the increase of motional impedance while the resistance R_e remains constant causes Q_e, the electrical Q, to decrease inversely with increasing B^2.

But as the frequency decreases, the motional impedance rises, reaching at f_h and again at f_l a maximum value of R_{es} which is usually several times the resistance R_e. Thus at these peaks the motional impedance, which at high frequencies was negligible compared with R_e, is now the major part of the total impedance. Suppose for simplicity that it comprises all of the speaker impedance. This time when B is varied and the motional impedance

[4] This should not be confused with the technique of mass control practiced by politicians and advertising people. In that context, the reactance is usually assumed to result from the equivalent of a compliance, and hence to decrease with signal frequency.

varies as B^2, then for a given acoustic power output the voltage across R_{es}, which is virtually the input voltage, will need to increase with increasing B. Summarizing, for a fixed acoustic power output, an increase of B will decrease the input voltage required at high frequencies, and increase the input voltage required at the impedance peaks. Also Q_e will decrease.

With a load impedance much larger than nominal, the criterion of performance of the amplifier becomes, not output power, but the undistorted output voltage on open circuit. This will always be larger than the undistorted output voltage at nominal load; how much larger will depend on the design of the amplifier.

Now if the Q_t required for a flat frequency response is identical with the Q_e of the loudspeaker, then if we ignore Q_a, the generator impedance R_g must be zero. Thus for a constant acoustic power output the same voltage will be required at the loudspeaker terminals at all frequencies, and all impedances, so that at the frequency f_h somewhat more maximum acoustic power is available than at higher frequencies.

If the Q_t required is less than Q_e, R_g will need to be negative, and for constant acoustic power and amplifier output voltage, at the junction of R_g and R_e in Fig. 4, will fall at f_h. But if the Q_t required is greater than Q_e, R_g will need to be positive, and the amplifier output voltage for constant acoustic power will rise at f_h. If the ratio of increase of voltage required is greater than the ratio of amplifier undistorted output voltages on open circuit to on-load, it is possible for less maximum acoustic power to be available in the region of f_h than at other frequencies in the useful band. But since low values of Q_e are normally associated with high efficiency, this is only likely to occur with high-efficiency, usually high-quality speakers. It should not cause trouble until Q_e is less than half Q_t, and even then the maximum acoustic power in most program material is less at frequencies below 100 Hz than around 400 Hz.

Thus there is a paradox that a highly efficient speaker may deliver less power around f_h than at higher frequencies, while a less efficient speaker delivers more. This will depend on the ratio of Q_e to Q_t and of amplifier undistorted output voltage off-load to on-load.

Related to this topic is the flattening of the impedance characteristic which is usually considered to be a good feature of vented boxes. Reference to Fig. 5, and comparison with Fig. 16, shows that, with the simplifying assumption that the resistive losses in the box and vent are negligible, the height of the impedance peak $R_e + R_{es}$ peaks at f_h and f_l and raise the minimum impedance at f_b. But this is incidental, and the relative heights are of little importance. Thus the idea of tuning the box so that the impedance peaks at f_h and f_l are equal, misses the real point. In the impedance curve of a loudspeaker in a box, the most useful information is not the values of the impedances, so long as box and vent damping is not too severe, but the values of the frequencies f_h, f_b, and f_l. Knowledge of these three frequencies alone enables a box alignment to be checked by Eqs. (105) and (106).

It should be clear that flatness of the impedance characteristic is no indication of flatness of acoustic response. Take as an analogy a coupled pair of tuned circuits. When the output voltage, or more exactly the transfer impedance, is maximally flat, the input impedance has two peaks. If one parameter is known, say the ratio of primary to secondary Q, the transfer impedance can be deduced from the input impedance, just as we do for loudspeakers in Eqs. (105) and (106). But a flat input impedance characteristic does not indicate a flat transfer impedance. In a loudspeaker, the impedance characteristic has greater peaks, whose height depends purely on the acoustic damping, though this contributes little to the overall system damping, and thus the overall frequency response.

XIII. EFFECTIVE REVERBERATION TIME

An objection sometimes made to the use of vented boxes is that the slope of attenuation beyond cutoff, 24 dB per octave, is much steeper than the 12 dB per octave of a speaker on an infinite baffle, and therefore the transient response is worse. In a low-pass filter, the ringing associated with steep attenuation slope is virtually removed by the use of Thompson or critically damped responses. But in high-pass filters such as are considered here, there is always some overshoot with filters of order two or more. To estimate its effect on a listener we use the concept of "effective reverberation time."

Imagine that we have a source of sound in a room which has built up a steady field. The source is then stopped. The sound in the room does not stop immediately, but dies away gradually. The time taken for the sound to decay is called the reverberation time, defined as the time taken for the sound pressure in the room to fall 60 dB from its original value. In small rooms the reverberation time will probably lie between 400 ms for a highly damped room to 1 s or more for a live one.

When the sound passes through two reverberant rooms in cascade, the law of the resulting overall reverberation time is not well establishd, but calculations on cascaded high-pass filters suggest that rms addition gives at least a guide. In any case it would appear that an added reverberation time of 200–300 ms should not appreciably color the reproduction.

When a transient is applied to a filter and it rings, the effect is perceived by the ear, or brain, as an extension of the transient event in time. Hence the expression "hang-over." To express the effect of the ringing then, an idea is borrowed from architectural acoustics, and the effective reverberation time of a filter is defined as the time taken, after a step function is applied, for the amplitude of the envelope of ringing to fall 60 dB below the amplitude of the original step function.

For the higher order filter functions, with two or more second-order factors, only the most lightly damped factor need be considered. For, by the time the ringing due to the most lightly damped factors is 60 dB down, the ringing due to the more heavily damped factors is negligible. This eases computation greatly.

Actually, at low frequencies the reverberation time defined above will be rather longer than the time the sound is perceived by the listener. To see why, we consult the much abused Fletcher–Munson curves [4, Fig. 12.11].

Suppose, for example, that the original sound is at 100-phon level. This is probably the maximum a system could reproduce, or a listener tolerate. Now at 50 Hz the threshold of hearing is 51 dB above reference level, that is, 49 dB below our arbitrary listening level. At 25 Hz the threshold of hearing is 67 dB above reference

Table IV. Reverberation times for various alignments.

Type of response	B_2	C_2	C_2	B_4	C_4	B_6	C_6	C_6	C_6
Q_t (for second order alignments)	0.707	1.000	1.414	—	—	—	—	—	—
k (for sixth order alighments)	—	—	—	—	—	1.000	0.600	0.414	0.268
Alignment numbers	—	—	—	5	8	15, 20, 26	17, 22	19, 24	25, 27
Time (in periods of cutoff frequency)	1.63	2.24	3.17	2.87	7.09	4.77	6.79	9.67	14.86
Time for 50 c/s cutoff (mS)	33	45	63	57	142	95	136	193	297

level, that is, only 33 dB below our arbitrary listening level. At 25 Hz, therefore, the effective reverberation time for the listener cannot be greater than the time in which the sound level falls 33 dB, i.e., about half the reverberation time as defined conventionally. Thus at low frequencies in general, the conventional definition based on a 60-dB fall in level yields a reverberation time rather longer than a listener will hear. (This is probably the reason for the observed increase in optimum reverberation time at low frequencies, see [4, Fig. 11.11].)

In a filter which cuts off sharply, the major ringing frequency will be close to the cutoff frequency. Also for a given shape of response curve the reverberation time can be expressed as a certain number of cycles of the cutoff frequency (see Table IV), i.e., the reverberation time increases with decreasing cutoff frequency. On the other hand, below, say, 50 Hz, its effect on the listener will decrease at approximately the same rate. Thus for all filters of a given response curve shape, the figure for 50 Hz should give a rough idea of the maximum reverberation time, as perceived by the listener.

Calculated reverberation times are given in Table IV. The first three alignments are of second order, corresponding to a loudspeaker on an infinite baffle. For these, the values of Q_t are shown. Note that the reverberation time, though low, doubles as Q_t increases from 0.707 to 1.414, that is, when the frequency response goes from maximally flat to a 4-dB peak. The times for 50-Hz cutoff are all below 200 ms, except for the last ($k = 0.268$), which is the very steepest.

It thus appears that a properly adjusted vented box, even with amplifier assistance (auxiliary filtering), need cause no perceptible coloration due to ringing. But it is important to emphasize that the adjustment must be correct. Table IV shows that the addition of a 4-dB peak to the response of a speaker on an infinite baffle can double the reverberation time. Being low in the first place it remains tolerable. But in the case of a vented box, particularly with an auxiliary filter, a doubled reverberation time would be more serious. Again, this emphasizes the importance of adequate damping (for correct value of Q_t) by the amplifier.

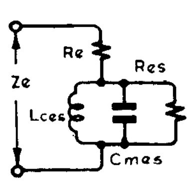


Fig. 15. Simplified equivalent electrical circuit of loudspeaker.

XIV. MEASUREMENT OF LOUDSPEAKER PARAMETERS

In earlier sections it was shown how the required response can be obtained from a loudspeaker and box if several parameters are known. The question remains, how are these parameters found?

Properly, this information should be available from the loudspeaker manufacturer. This is particularly important for equipment produced in quantity, where it is important to know not only the mean values but also the tolerances. However, in the absence of published figures, or to check them, the following procedure will provide the information.

Procedures for measuring Q are given in [2, p. 13], but the method used seems too laborious and inaccurate. The method outlined hereafter can be understood by considering Figs. 15 and 16. Figure 15 is derived from Fig. 4; only this time we omit the vented box and we ignore L_e and R_{sh} which take effect at much higher frequencies. Now

$$Q_a = \omega_s C_{mes} R_{es} \tag{91}$$

$$Q_e = \omega_s C_{mes} R_e. \tag{92}$$

These quantities, defined earlier in Eqs. (71) and (72) in terms of the acoustic equivalent circuit, are defined here in terms of the electrical equivalent circuit. We define r_0 as the ratio of the impedance at resonance, $R_{es} + R_e$, to the dc resistance of the voice coil R_e. Now we take another arbitrary impedance which is presented at two other frequencies f_1 and f_2 on the flanks of the curve, and we call its ratio to the dc resistance r_1. Then

$$f_1 f_2 = f_s^2. \tag{93}$$

Physically, this means that the curve is symmetrical on a logarithmic frequency scale. In experimental work it provides a handy check. Now we can find

$$Q_a = [f_s/(f_2-f_1)][(r_0^2-r_1^2)/(r_1^2-1)]^{1/2} \tag{94}$$

and

$$Q_e = Q_a/(r_0-1). \tag{95}$$

If additionally we choose r_1 such that

$$r_1 = \sqrt{r_0} \tag{96}$$

then Eq. (94) is simplified to

$$Q_a = \sqrt{r_0}\, f_s/(f_2-f_1). \tag{97}$$

The interesting feature of these expressions is that they involve no approximations, and thus hold for all values of Q. Furthermore around the value $\sqrt{r_0}$ the curve has its greatest slope. Thus the frequencies f_1 and f_2 can be

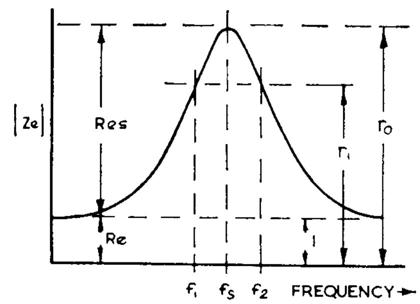


Fig. 16. Typical impedance curve of loudspeaker, modulus of Z_e in Fig. 15.

found most accurately. This is especially important since the calculation involves a comparatively small difference between large numbers $f_2 - f_1$.

Usually Q_a takes account of the acoustic resistances in the loudspeaker. But if the voice coil has a short-circuited turn by accident or design, e.g., an aluminum former, this will appear in Q_a, even though its physical nature is similar to Q_e. (But eddy current losses in the pole piece or front plate appear in R_{sh}.)

Fig. 17 shows the test circuit. V is a voltmeter of impedance much higher than the loudspeaker. Throughout the readings, the generator is adjusted so that the reading of V is constant. The value is not of great importance, but a standard test figure is one volt. The accuracy of this voltmeter is not important so long as it is independent of frequency. A is an ac ammeter which reads the current into the speaker with the fixed voltage across its terminals. Again, since we are interested only in the shape of the impedance curve, the absolute accuracy of this instrument is not important so long as the meter reading is linear. However, to set the relative current due to R_e, first we measure R_e with dc on a Wheatstone bridge, and then a calibrating resistor R_c of similar value. Connecting R_c to the test terminals and applying the standard test voltage at say, f_s, a current value I_c is found on the ammeter A. Then the current I_e which corresponds to R_e is found by

$$I_e = I_c R_c / R_e. \tag{98}$$

Now the loudspeaker is suspended in air as far from reflecting surfaces as is practical and connected to the test terminals instead of R_c. The generator is adjusted to the speaker resonant frequency f_s, indicated by minimum current I_o. Thus r_o is found:

$$r_o = I_e / I_o. \tag{99}$$

Now the current $\sqrt{(I_e I_o)}$ is found corresponding to the ratio $\sqrt{r_o}$ and the frequencies either side of resonance, where this current value is read. These are f_1 and f_2 and they should be read to as close an accuracy as the test gear will allow. Eq. (93) provides a check on the method, and Eqs. (97) and (95) give Q_a and Q_e.

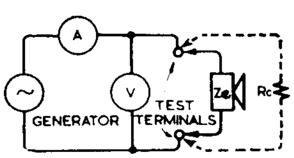


Fig. 17. Test circuit schematic for measurement of loudspeaker parameters.

The next problem is to find the value of V_{as}, the volume of air equivalent to the loudspeaker compliance. For this, the loudspeaker is placed in a totally enclosed unlined box whose internal volume V_b is known, remembering that allowance must be made for bracing and the volume displaced by the speaker. It is important that this box be free of air leaks. If these occur we will read part of the curve of Fig. 5, around f_h. Thus care should be taken, not only in the construction of the box and in the mounting of the speaker, but also in the way the speaker leads are taken through the walls of the box. Solid terminals are preferred.

Another precaution may be necessary. In Figs. 15 and 16, from which we derived Eqs. (93), (94), (95), and (97), we assumed that the effect of the inductance L_e is negligible. In fact, L_e interacts with the parallel combination of L_{ces} and C_{mes} to produce a series resonance at f_n in Fig. 5, where the nominal impedance is measured. If this frequency, usually 400–600 Hz, is well above the speaker resonance f_s, so that there is little disturbance of the curve at f_2 of Fig. 16, the accuracy of the measurements will be unaffected. But if f_s is above 150 Hz, which can occur with small speakers and becomes even more likely when the speaker is placed in the box for the last test, the likelihood of inaccurate results increases.

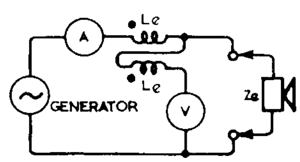


Fig. 18. Modification of Fig. 17 to cancel effect of loudspeaker inductance L_e.

This could be avoided by connecting in the circuit of Fig. 18 a bifilar inductance whose value L_e in each half is equal to the inductance of the voice coil. It is preferable, and not difficult, to wind this with an air core. In measuring L_e of the loudspeaker, it is important to measure it at a frequency well away from f_n, say, 10 kHz. Also it is important to measure it as an inductance in parallel with a resistance (D or tan δ scale, *not* the Q scale of a bridge), for the Q of the inductance at 10 kHz is usually of the order of one which can lead to serious error if the measurement is made as an inductance in series with a resistance. With a high-impedance voltmeter V, error due to series resistance of the inductor should be negligible.

If the new resonant frequency in the closed box f_{sc} is found, the ratio of volume is usually given as

$$V_{as}/V_b = (f_{sc}/f_{sa})^2 - 1 \tag{100}$$

where f_{sa} is the resonant frequency of the speaker in air which we previously called f_s. However, this expression ignores the change in the acoustic mass M_{as} of 1.05 to 1.25 times which results from placing the speaker in the box. A more accurate method is to repeat the previous procedures for finding Q_e. Then if we call Q_{ea} and Q_{eo} the values of Q_e measured in air and in the closed box, respectively, then

$$V_{as}/V_b = [(f_{sc}Q_{ec})/(f_{sa}Q_{ea})] - 1. \tag{101}$$

Also the ratio of the acoustic masses in air and in the closed box

$$M_{asa}/M_{asb} = f_{sc}Q_{ca}/f_{sa}Q_{ec} \qquad (102)$$

should lie between 0.8 and 0.95.

With V_b known, V_{as} can be calculated. The size of V_b is not critical, but should not be too large, otherwise the ratio f_{sc}/f_{sa} becomes close to unity, and the accuracy of the V_{as}/V_b calculation falls. This can be seen from Eq. (100). Finally the values of f_{sa} and Q_{ea} are adjusted to take account of the change in M_{as} when the speaker is placed in the box. Thus

$$f_{sb} = f_{sa}(M_{asa}/M_{asb})^{1/2} \qquad (103)$$

$$Q_{eb} = Q_{sa}/(M_{asa}/M_{asb})^{1/2}. \qquad (104)$$

Thus the efficiency η_{ob} can be calculated from Eq. (77). This gives the result, rather surprising at first sight, that the electroacoustic conversion efficiency of a loudspeaker in the piston range can be calculated from electrical measurements alone.

The following alternative method is useful, particularly when the loudspeaker has to be placed in a box whose size is already determined or as a final check on a previously calculated box, or again if it becomes too difficult to seal the loudspeaker in the test box.[5]

First the vent, if adjustable, is made to resonate with the box somewhere near the speaker resonant frequency, but this is not very important. Then the three frequencies f_l, f_b, and f_h of Fig. 5 are found as accurately as possible. Special care is needed in reading f_b as the curve has a flat bottom.

From these readings we find f_{sb}, the resonant frequency of the speaker when mounted in the box,

$$f_{sb} = f_h f_l / f_b \qquad (105)$$

and the compliance ratio C_{as}/C_{ab}, i.e.,

$$V_{as}/V_b = (f_h^2 - f_b^2)(f_b^2 - f_l^2)/f_h^2 f_l^2. \qquad (106)$$

With the speaker resonant frequency in air f_{sa} already known and f_{sb} known from Eq. (105), we find the mass ratio M_{asa}/M_{asb} from Eq. (103), and then Q_{eb} from Eq. (104). Q_a is adjusted to Q_{ab} in a similar manner. By reference to Table I and Fig. 7, a suitable alignment can be found, thus setting the final values of f_b and Q_t. Note that Q_t is due to the parallel combination of 1) Q_{ab} and 2) Q_{eb} modified by the amplifier.

To estimate the value of Q_b, the "Q of the box and vent circuit," we measure I_b, the current through the speaker at f_b, with the input voltage held constant as before. Then

$$Q_b = (\omega_b/\omega_s)(C_{ab}/C_{as})[(1/Q_e) + (1/Q_a)][(I_b - I_o)/(I_e - I_b)]. \qquad (107)$$

[5] Experience gained since the writing of this paper shows that accurate results are more easily obtained with this second method. Using a vented box is especially preferred if the speaker being measured has a low resonant frequency and if the testing box is fairly small. In such cases, small leaks in the "totally enclosed" box or around the loudspeaker pad ring can produce a virtual vent which produces the familiar twin peaks of loudspeaker impedance. But if the lower peak is below the limit of measurement, say, below 10 or 15 Hz, it could easily happen that the remaining upper peak would be taken as the single peak of a closed-box system with dire results.

Note that because the difference between I_e and I_b will be small, the readings must be taken carefully.

Comparing Eq. (107) with Eq. (89), it can be seen that

$$(A_b)_{max} = 1/\{1 + [Q_a Q_e/Q_t(Q_a + Q_e)][(I_e - I_b)/(I_b - I_o)]\}. \qquad (108)$$

This greatly simplifies the estimation of $(A_b)_{max}$.

A worked example of this method is given in the Appendix.[6]

XV. EXPERIMENTAL WORK

When the work was started from which this paper derived, it was necessary first to find the parameters for a number of loudspeakers. To date about fifty have been measured. In the case of one speaker, the effect of a number of modifications was observed; in the rest, usually one and occasionally two or three samples have been checked. The results obtained give confidence in the method. For example, from the readings and knowing other parameters, it is possible to calculate the flux density, and the values obtained give good correlation with readings on a flux meter. Changes of parameters during production can also be detected.

Some generalizations from the results have been mentioned earlier. For example, it was found that Q_a varies between about 3 and 10, which is high compared with the Q_t values of 0.2 to 0.6 required in Table I. Thus it was apparent that acoustic resistance usually has little effect on the damping of a speaker in a well-designed system. Values of Q_e varied from 0.2 to 0.5 in the case of high-quality speakers, through 0.5 to 1.0 in the better commercial grades of speakers, to 2 and even 3 in the case of some low-priced speakers.

Similarly efficiencies, for radiation from one side of an infinite baffle, ranged from −24 dB (0.4%) for low-priced speakers through −20 dB (1%) for medium-grade to −14 dB (4%) for high-quality speakers.

However, one must resist the tempting generalization that it is possible to rate the overall quality of a speaker by its Q_e or even its efficiency. For example, if efficiency is made higher and Q_e lower by reducing the cone mass M_{ms}, trouble with "break up" may result at middle frequencies. In fact while the best 8-in speaker tested had a Q_e of 0.33, there was one sample with good clean response at high frequencies with a high Q_e of 1.7 and another with Q_e below 1 which was less acceptable. It must be remembered that these readings, and the paper in general, are concerned only with low-frequency performance.

As a result of the design theory, a number of boxes have been made. In the absence of reliable measurements of sound pressure, all that can be said is that they gave a good improvement in clean low-frequency response, and that the cutoff frequencies are near the predicted values. Some particularly gratifying results have been obtained

[6] Experimental work, using the above method indicates that in practical boxes Q_b is often of the order of 10. This difference from the calculated values of 30 or more may be due to frictional losses in the timber. It is shown in Section XI that when Q_b is 10, the frequency response error is still only 1 to 2 dB. However, if there are sufficient air leaks, or if the cavity damping is excessive, as when the box is completely stuffed with underfelt, Q_b can fall below 5.

with 5-in speakers in modest boxes with response down to 80 Hz.

XVI. CONCLUSION

The work described herein was begun as an advanced development project in an attempt to obtain good low-frequency response from loudspeakers in small boxes. Unfortunately, no "revolutionary concept" was uncovered that offers something for nothing. On the other hand, it has provided a reasonably precise method of design that was previously lacking.

In general, a system with good flat response down to a predictable cutoff frequency can be designed, if the necessary parameters Q_e (and Q_a), V_{as}, and f_s are known for the loudspeaker. The box volume is closely proportional to the inverse square of cutoff frequency, which can be varied over a wide range. The output impedance R_g of the amplifier has a large effect in controlling the response, especially at f_h, the higher frequency of maximum impedance. Whether R_g needs to be positive, zero, or negative depends on the type of alignment and the Q parameters of the speaker. On the evidence available, acoustic resistance damping of the vent has no advantage, and is wasteful of box volume or bandwidth.

The advantages accruing from a predictable design include the possibility of optimum design of "rumble" filters. At frequencies below cutoff where negligible acoustic output is produced, these relieve the amplifier and loudspeaker of high signal amplitudes and thus minimize an annoying source of intermodulation distortion. Carried a step further, the use of auxiliary electrical filters makes it possible to trade box volume for low-frequency power capability of the amplifier.

Another way of reducing box volume is to increase the mass of the loudspeaker cone. But since this also reduces efficiency, it may be considered as a further example of trading amplifier size for box size, only this time the amplifier must deliver increased power over the whole audio spectrum. Again, the box volume may be reduced if a smaller diameter loudspeaker is used. The danger here is that the speaker excursion increases, but it is a good solution if the speaker is capable of a long linear excursion, or if the power output and/or low-frequency response is restricted.

The size of the magnet, or more precisely the flux density B, has a great influence on performance. Both efficiency, hence acoustic output, and Q_e vary with B^2; so it is clear that the saving of pennies on a smaller magnet can be poor economy.

The parameters needed for vented-box design can be measured with normal electrical measuring equipment together with a test box of known net internal volume. Nevertheless it is suggested to loudspeaker manufacturers that it is in their interest, as well as the user's, to publish typical values of Q_e, Q_a, V_{as}, and x_{max}, as well as f_s. These parameters are more useful to the system designer than, for example, flux density or total flux. Their publication would help ensure that the manufacturer's product is used to the best possible advantage.

The totally enclosed box has been mentioned only in passing, since it is well covered in [2]. But it should be noted that if a totally enclosed box is chosen with the same volume as that of alignment no. 5, the cutoff frequency is 1.55 times higher. With smaller boxes, the advantage decreases, though with practical sizes it is still appreciable. With larger totally enclosed boxes, the cutoff frequency can never fall below f_s, while the Chebyshev vented box alignments can extend the response considerably below f_s.

The greatest advantage of a vented box over an infinite baffle is the reduction of loudspeaker excursion, permitting higher power output or lower distortion. To this advantage, the present paper adds, it is hoped, a greater flexibility in design. The only apparent disadvantage of a vented box is in the transient response, but in fact the ringing is only perceptible with a misadjusted alignment. With proper adjustment, the effective reverberation time, though longer than that of a properly adjusted infinite baffle, is not long enough to appreciably color the sound in the listening room.

Finally, it is emphasized again that the acoustic response is due to the combination of speaker plus box plus amplifier as an integrated whole.

APPENDIX: WORKED EXAMPLE

This refers to a purely imaginary speaker, the readings being chosen to simplify the calculations. However, the readings would be typical of a medium-quality 8-in speaker.

Measurement of Speaker Parameters Q_a, Q_e, V_{as}, and f_s

With a Wheatstone bridge we find

dc resistance of speaker $R_e = 4.00$ ohms
dc resistance of calibrating resistor $R_c = 5.00$ ohms.

Now we place R_c in the test circuit of Fig. 17 and find that when V reads 1 volt,

$$I_c = 180 \text{ mA}.$$

Now

$$I_cR_c = 0.180\times 5.00 = 0.900.$$

Since this is 10% below the observed reading of 1 volt, one or both of the meters is inaccurate, but this is unimportant so long as their readings are constant with frequency and the reading of ammeter A is linear.

Then from Eq. (98),

$$I_e = I_cR_c/R_e = (0.180\times 5.00)/4.00 = 225 \text{ mA}.$$

We now suspend the loudspeaker in air as far from reflecting surfaces as possible and read the minimum current I_0 which is 25 mA at 55.0 Hz (f_{sa}, the speaker resonant frequency in air).

Then from Eq. (99),

$$r_0 = I_e/I_0 = 225/25 = 9$$

$$\sqrt{r_0} = \sqrt{9} = 3$$

$$\sqrt{(I_0I_e)} = \sqrt{(225\times 25)} = 75 \text{ mA}.$$

With the voltmeter V reading a constant 1 volt, the ammeter A reads 75 mA at 44.0 and 68.75 Hz.

First we use this reading to check $f_{sa} = \sqrt{(44.0\times 68.75)}$ from Eq. (93) = 55.0 Hz as before. Then from Eq. (97),

$$Q_a = f_a\sqrt{r_0}/(f_2-f_1) = (55\times 3)/(68.75-44) = 6.67$$

and from Eq. (95),

$$Q_e = Q_a/(r_o - 1) = 6.67/(9-1) = 0.833.$$

The speaker is now placed in a vented box whose net volume is 1000 in^3 and we read the frequencies defined in Fig. 5,

$$f_h = 100\,\text{Hz}; \quad f_b = 60\,\text{Hz}; \quad f_l = 30\,\text{Hz}.$$

Then from Eq. (105),

$$f_{sb} = f_h f_l / f_b = (100 \times 30)/60 = 50\,\text{Hz}$$

and from Eq. (106),

$$V_{as}/V_b = (f_h^2 - f_b^2)(f_b^2 - f_l^2)/f_h^2 f_l^2.$$

Computation is easier if we rewrite Eq. (106) as

$$V_{as}/V_b = (f_h + f_b)(f_h - f_b)(f_b + f_l)(f_b - f_l)/f_h^2 f_l^2$$

i.e.,

$$\begin{aligned} V_{as}/V_b &= (100+60)(100-60)(60+30)(60-30)/100^2 \times 30^2 \\ &= (160 \times 40 \times 90 \times 30)/(100 \times 30 \times 100 \times 30) \\ &= 1.92 \end{aligned}$$

i.e.,

$$V_{as} = 1.92 \times 1000 = 1920\,\text{in}^3.$$

In the vented box, the speaker resonant frequency has dropped $f_{sb}/f_{sa} = 50/55 = 0.909$ times. Thus from Eq. (103),

$$M_{asa}/M_{asb} = (0.909)^2 = 0.826$$

and from Eq. (104),

$$Q_{ab} = 6.67/0.909 = 7.33$$

while

$$Q_{eb} = 0.833/0.909 = 0.917.$$

At f_b the current I_b was read as 220 mA. Then from Eq. (107), the Q of the box plus vent

$$\begin{aligned} Q_b &= (f_b/f_s)(C_{ab}/C_{as})[(Q_a+Q_e)/Q_a Q_e][(I_b - I_o)/(I_e - I_b)] \\ &= [60 \times (7.333 + 0.917) \times (220-25)]/[50 \times 1.92 \times 7.33 \times 0.917 \times (225-220)] \\ &= (60 \times 8.25 \times 195)/(50 \times 1.92 \times 7.33 \times 0.917 \times 5) \\ &= 29.9. \end{aligned}$$

From Eq. (108) the maximum box loss in the quasi-Butterworth alignment described below, where $Q_t = 0.347$, is

$$\begin{aligned} (A_b)_{max} &= 1/\{1 + [Q_a Q_e / Q_t (Q_a + Q_e)][(I_e - I_b)/(I_b - I_o)]\} \\ &= 1/\{1 + (7.33 \times 0.917 \times 5)/(0.347 \times 8.25 \times 195)\} \\ &= 1/1.060 \end{aligned}$$

which is equivalent to 0.5 dB.

Efficiency η from Eq. (77)

$$\begin{aligned} \eta_{ob} &= 8.0 \times 10^{-12} f_s^3 V_{as}/Q_e \\ &= (8.0 \times 50^3 \times 1920)/(10^{12} \times 0.917) \\ &= 2.09 \times 10^{-3} \end{aligned}$$

which is equivalent to −26.6 dB in a box, or −23.6 dB on an infinite baffle (i.e., a true infinite baffle, not a totally enclosed medium-sized box which gives the same efficiency as a vented box), or −20.6 dB on a true infinite baffle, taking into account radiation from both front and back.

Thus if the speaker is mounted in a box and fed a 5-watt amplifier, the acoustic power output will

$$W_{ao} = \eta_{ob} W_{ei} = 5 \times 2.09 \times 10^{-3} = 0.0104\ \text{Wa}$$

If we assume a listening room of 16 × 12½ × 10 = 2000 ft^3, then from [4, p. 418, Fig. 11.12] an acoustic power of 0.003 watt provides +80-dB intensity level. Our output is 10.4/3 times, i.e., 5.4 dB greater than this; therefore the system is capable of a peak +85-dB intensity level.

Peak Excursion x_{pk}

We assume an alignment where the box is tuned to the same frequency as the loudspeaker, i.e., 50 Hz. This is typical of Butterworth alignments. Then the fixed part of the expression for x_{pk} in Eq. (84) is

$$(1.31 \times 10^5 \times \sqrt{W_{ao}})/f_b^2 S_d.$$

Now if the effective piston diameter is 7 in, i.e.,

$$S_d = \pi \times 3.5^2 = 38.5\,\text{in}^2$$

then the expression becomes

$$1.31 \times 10^5 \times \sqrt{0.104}/(50^2 \times 38.5) = 0.139\,\text{in}.$$

Now the maximum value of the frequency-sensitive expression for a vented box in the useful band (above f_b) in Fig. 10 is approximately one quarter. Thus

$$x_{pk} = 0.139/4 = \pm 0.035\,\text{in}$$

compared with ±0.098 in in a totally enclosed box (infinite baffle).

Box Design

First suppose we wish to obtain the best results with the original 1000-in^3 box. Allowing 10% for the bracing and volume displaced by the speaker, the optimum inside dimensions would be $\sqrt[3]{1100} \times (0.8, 1.0, 1.25)$ in, i.e., $8.28 \times 10.33 \times 12.9$ in, say 8¼ × 10¼ × 13 in. This would need to be checked in case the original assumption of 10% was incorrect. Assuming that the dimensions are

Table V. Computation of three Butterworth alignments for imaginary speaker.

Type of alignment	QB_3	B_4	B_5	B_6(i)
C_{as}/C_{ab}	1.92	1.414	1.000	2.732
V_b (cubic inches)	1000	1358	1920	704
Box: Height (in.)	13	14	16	11½
Box: Width (in.)	10¼	11½	13	9
Box: Depth "d" (in.)	8¼	9	10	7½
Cutoff frequency f_3 (c/s)	58.5	50	50	50
Box frequency f_b (c/s)	54.7	50	50	50
L_v/S_v (in.$^{-1}$)	1.56	1.37	0.97	2.65
S_v (in.^{2})	7.69	10.07	16.25	4.50
Vent height "l" (in.)	¾	⅞	1¼	½
Q_t	.347	.383	.447	.299
$(Q_e)_{total}$	.364	.404	.476	.312
R_g/R_e	−.600	−.560	−.481	−.660

then in a box similar to Fig. 9, the width of will be $10\frac{1}{4}$ in. The length of the tunnel will be $\frac{3}{4}$ in, together with two thicknesses of timber (say in each) plus a $\frac{1}{2}$-in square stiffener on the top rear edge of the shelf, giving a total tunnel length of $9\frac{3}{4}$ in.

The simplest alignment for $C_{as}/C_{ab} = 1.92$ is a third-order quasi-Butterworth between alignments no. 4 and 5. From Fig. 7 (b),

$$f_3/f_s = 1.17, \text{ thus } f_3 = 50 \times 1.17 = 58.5 \text{ Hz}$$

$$f_3/f_b = 1.07, \text{ thus } f_b = 58.5/1.07 = 54.7 \text{ Hz.}$$

Thus

$$\omega_b^2 = 1.18 \times 10^5$$

and for the tunnel, from Eq. (61),

$$\begin{aligned}(L_v/S_v)_{required} &= 1.84 \times 10^8/\omega_b^2 V_b \\ &= 1.84 \times 10^8/1.18 \times 10^5 \times 10^3 \\ &= 1.56 \text{ in}^{-1}.\end{aligned}$$

Now if the tunnel height $l = \frac{3}{4}$ in, then area

$$S_v = 10\tfrac{1}{4} \times \tfrac{3}{4} = 7.69 \text{ in}^2$$

and

$$\begin{aligned}(L_v/S_v)_{end} &= 0.958/\sqrt{S_v} \\ &= 0.958/\sqrt{7.69} \\ &= 0.34 \text{ in}^{-1}\end{aligned}$$

$$\begin{aligned}(L_v/S_v)_{tunnel} &= 9.75/7.69 \\ &= 1.27 \text{ in}^{-1}.\end{aligned}$$

Thus

$$(L_v/S_v)_{available} = 1.61 \text{ in}^{-1}$$

which is about as close as can be obtained with the tolerances on the small dimension ($\frac{3}{4}$ in) of l.

Amplifier Output Impedance R_g

Now by interpolation,

$$Q_t = 0.347$$

and since $Q_{ab} = 7.33$, $Q_{eb} = 0.917$, and from Eq. (70),

$$1/Q_t = 1/Q_a + 1/Q_e(1 + R_g/R_e).$$

Thus

$$1/0.347 = 1/7.33 + 1/0.917(1 + R_g/R_e).$$

Hence

$$R_g/R_e = -0.60.$$

Notes

1) $(L_v/S_v)_{end}$ is small compared with $(L_v/S_v)_{tunnel}$, and since the vent area is already small compared with the piston area, a simple hole in the front panel would be quite impractical as a vent. Its area would need to be about 1 in^2.

2) The dimension l ($\frac{3}{4}$ in) is fairly critical.

3) Q_a has little effect on Q_t. The negative impedance required is fairly high but quite practical.

For comparison three Butterworth alignments have also been computed for this imaginary speaker so that the effect of amplifier filtering can be assessed (Table V). All three have cutoff frequencies of 50 Hz. But while B_4 has no filtering, B_5 has a simple CR filter which is -3 dB at 50 Hz ($CR = 3180$ μs), and B_6 has a peak 6 dB high at 53.5 Hz before it falls off at the rate of 12 dB per octave ($y = -1.732$, $f_{aux} = 50$ Hz).

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