# Closed-Box Loudspeaker Systems
# Part I: Analysis

**RICHARD H. SMALL**

*School of Electrical Engineering, The University of Sydney*
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The closed-box loudspeaker system is effectively a second-order (12 dB/octave cutoff) high-pass filter. Its low-frequency response is controlled by two fundamental system parameters: resonance frequency and total damping. Further analysis reveals that the system electroacoustic reference efficiency is quantitatively related to system resonance frequency, the portion of total damping contributed by electromagnetic coupling, and total system compliance; for air-suspension systems, efficiency therefore effectively depends on frequency response and enclosure size. System acoustic power capacity is found to be fundamentally dependent on frequency response and the volume of air that can be displaced by the driver diaphragm; it may also be limited by enclosure size. Measurement of voice-coil impedance and other mechanical properties provides basic parameter data from which the important low-frequency performance capabilities of a system may be evaluated.

## GLOSSARY OF SYMBOLS

$B$ magnetic flux density in driver air gap
$c$ velocity of sound in air ($=345$ m/s)
$C_{AB}$ acoustic compliance of air in enclosure
$C_{AS}$ acoustic compliance of driver suspension
$C_{AT}$ total acoustic compliance of driver and enclosure
$C_{MEC}$ electrical capacitance representing moving mass of system ($=M_{AC}S_D^2/B^2l^2$)
$e_g$ open-circuit output voltage of source (Thevenin's equivalent generator for amplifier output port)
$f$ natural frequency variable
$f_C$ resonance frequency of closed-box system
$f_{CT}$ resonance frequency of driver in closed, unfilled, unlined test enclosure
$f_S$ resonance frequency of unenclosed driver
$G(s)$ response function
$k_x$ displacement constant
$k_P$ power rating constant
$k_\eta$ efficiency constant
$l$ length of voice-coil conductor in magnetic gap
$L_{CET}$ electrical inductance representing total system compliance ($=C_{AT}B^2l^2/S_D^2$)
$M_{AC}$ acoustic mass of driver in enclosure including air load
$M_{AS}$ acoustic mass of driver diaphragm assembly including air load
$P_{AR}$ displacement-limited acoustic power rating
$P_{ER}$ displacement-limited electrical power rating
$P_{E(max)}$ thermally-limited maximum input power
$Q$ ratio of reactance to resistance (series circuit) or resistance to reactance (parallel circuit)
$Q_{EC}$ $Q$ of system at $f_C$ considering electrical resistance $R_E$ only

| | |
|---|---|
| $Q_{ES}$ | $Q$ of driver at $f_S$ considering electrical resistance $R_E$ only |
| $Q_{MC}$ | $Q$ of system at $f_C$ considering system non-electrical resistances only |
| $Q_{MS}$ | $Q$ of driver at $f_S$ considering driver non-electrical resistances only |
| $Q_{TC}$ | total $Q$ of system at $f_C$ including all system resistances |
| $Q_{TCO}$ | value of $Q_{TC}$ with $R_g = 0$ |
| $Q_{TS}$ | total $Q$ of driver at $f_S$ considering all driver resistances |
| $R_{AB}$ | acoustic resistance of enclosure losses caused by internal energy absorption |
| $R_{AS}$ | acoustic resistance of driver suspension losses |
| $R_E$ | dc resistance of driver voice coil |
| $R_{ES}$ | electrical resistance representing driver suspension losses ($=B^2l^2/S_D^2R_{AS}$) |
| $R_g$ | output resistance of source (Thevenin's equivalent resistance for amplifier output port) |
| $s$ | complex frequency variable ($=\sigma + j\omega$) |
| $S_D$ | effective surface area of driver diaphragm |
| $T$ | time constant ($=1/2\pi f$) |
| $U_O$ | system output volume velocity |
| $V_{AB}$ | volume of air having same acoustic compliance as air in enclosure ($=\rho_0 c^2 C_{AB}$) |
| $V_{AS}$ | volume of air having same acoustic compliance as driver suspension ($=\rho_0 c^2 C_{AS}$) |
| $V_{AT}$ | total system compliance expressed as equivalent volume of air ($=\rho_0 c^2 C_{AT}$) |
| $V_B$ | net internal volume of enclosure |
| $V_D$ | peak displacement volume of driver diaphragm ($=S_D x_{max}$) |
| $x_{max}$ | peak linear displacement of driver diaphragm |
| $X(s)$ | displacement function |
| $Z_{VC}(s)$ | voice-coil impedance function |
| $\alpha$ | compliance ratio ($=C_{AS}/C_{AB}$) |
| $\gamma_B$ | ratio of specific heat at constant pressure to that at constant volume for air in enclosure |
| $\eta_0$ | reference efficiency |
| $\rho_0$ | density of air ($=1.18$ kg/m$^3$) |
| $\omega$ | radian frequency variable ($=2\pi f$) |

## 1. INTRODUCTION

### Historical Background

The theoretical prototype of the closed-box loudspeaker system is a driver mounted in an enclosure large enough to act as an infinite baffle [1, Chap. 7]. This type of system was used quite commonly until the middle of this century.

The concept of the modern air-suspension loudspeaker system was established in a U.S. patent application of 1944 by Olson and Preston [2], [3], but the system was not widely introduced until high-fidelity sound reproduction became popular in the 1950's.

A compact air-suspension loudspeaker system for high-fidelity reproduction was described by Villchur [4] in 1954. Several more papers [5], [6], [7] set out the basic principle of operation but caused a spirited public controversy [8], [9], [10]. Unfortunately, some of the confusion established at the time still remains, particularly with regard to the purpose and effect of materials used to fill the enclosure interior. A recent attempt to dispell this confusion [11] seems to have reduced the level of controversy, and the fundamental validity of the air-suspension approach has been amply proved by its proliferation.

### Technical Background

Closed-box loudspeaker systems are the simplest of all loudspeaker systems using an enclosure, both in construction and in analysis. In essence, they consist of an enclosure or box which is completely closed and airtight except for a single aperture in which the driver is mounted.

The low-frequency output of a direct-radiator loudspeaker system is completely described by the acoustic volume velocity crossing the enclosure boundaries [12]. For the closed-box system, this volume velocity is entirely the result of motion of the driver cone, and the analysis is relatively simple.

Traditional closed-box systems are made large so that the acoustic compliance of the enclosed air is greater than that of the driver suspension. The resonance frequency of the driver in the enclosure, i.e., of the system, is thus determined essentially by the driver compliance and moving mass.

The air-suspension principle reverses the relative importance of the air and driver compliances. The driver compliance is made very large so that the resonance frequency of the system is controlled by the much smaller compliance of the air in the enclosure in combination with the driver moving mass. The significance of this difference goes beyond the smaller enclosure size or any related performance improvements; it demonstrates forcibly that the loudspeaker driver and its enclosure cannot be designed and manufactured independently of each other but must be treated as an inseparable *system*.

In this paper, closed-box systems are examined using the approach described in [12]. The analysis is limited to the low-frequency region where the driver acts as a piston (i.e., the wavelength of sound is longer than the driver diaphragm circumference) and the enclosure is active in controlling the system behavior.

The results of the analysis show that the important low-frequency performance characteristics of closed-box systems of both conventional and air-suspension type are directly related to a small number of basic and easily-measured system parameters.

The analytical relationships impose definite quantitative limits on both small-signal and large-signal performance of a system but, at the same time, show how these limits may be approached by careful system adjust-

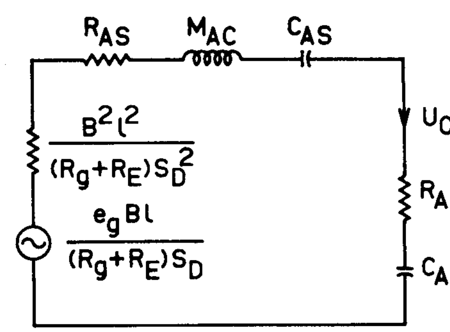


Fig. 1. Acoustical analogous circuit of closed-box loudspeaker system (impedance analogy).

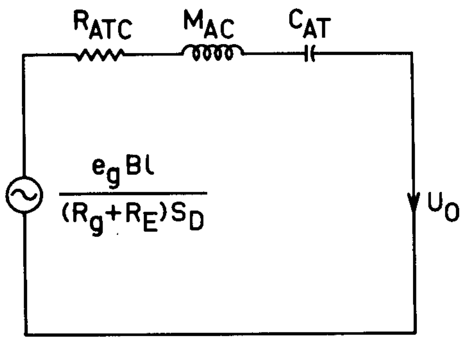


Fig. 2. Simplified acoustical analogous circuit of closed-box loudspeaker system.

ment. The same relationships lead directly to methods of synthesis (system design) which are free of trial-and-error procedures and to simple methods for evaluating and specifying system performance at low frequencies.

## 2. BASIC ANALYSIS

The impedance-type acoustical analogous circuit of the closed-box system is well known and is presented in Fig. 1. In this circuit, the symbols are defined as follows.

- $B$ Magnetic flux density in driver air gap.
- $l$ Length of voice-coil conductor in magnetic field of air gap.
- $e_g$ Open-circuit output voltage of source.
- $R_g$ Output resistance of source.
- $R_E$ Dc resistance of driver voice coil.
- $S_D$ Effective projected surface area of driver diaphragm.
- $R_{AS}$ Acoustic resistance of driver suspension losses.
- $M_{AC}$ Acoustic mass of driver diaphragm assembly including voice coil and air load.
- $C_{AS}$ Acoustic compliance of driver suspension.
- $R_{AB}$ Acoustic resistance of enclosure losses caused by internal energy absorption.
- $C_{AB}$ Acoustic compliance of air in enclosure.
- $U_O$ Output volume velocity of system.

By combining series elements of like type, this circuit can be simplified to that of Fig. 2. The total system acoustic compliance $C_{AT}$ is given by

$$C_{AT} = C_{AB}C_{AS}/(C_{AB} + C_{AS}), \tag{1}$$

and the total system resistance, $R_{ATC}$, is given by

$$R_{ATC} = R_{AB} + R_{AS} + \frac{B^2l^2}{(R_g + R_E)S_D^2}. \tag{2}$$

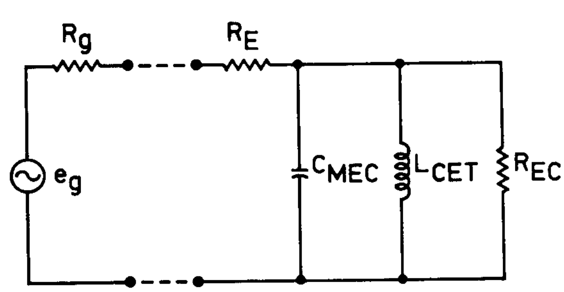


Fig. 3. Simplified electrical equivalent circuit of closed-box loudspeaker system.

The electrical equivalent circuit of the closed-box system is formed by taking the dual of the acoustic circuit of Fig. 1 and converting each element to its electrical equivalent [1, Chapter 3]. Simplification of this circuit by combining elements of like type results in the simplified electrical equivalent circuit of Fig. 3. This circuit is arranged so that the actual voice-coil terminals are available. In Fig. 3, the symbols are given by

$$C_{MEC} = M_{AC}S_D^2/B^2l^2, \tag{3}$$

$$L_{CET} = C_{AT}B^2l^2/S_D^2, \tag{4}$$

$$R_{EC} = \frac{B^2l^2}{(R_{AB} + R_{AS})S_D^2}. \tag{5}$$

The circuits presented above are valid only for frequencies within the driver piston range; the circuit elements are assumed to have values which are independent of frequency within this range. As discussed in [12], the effects of the voice-coil inductance and the resistance of the radiation load are neglected.

To simplify the analysis of the system and the interpretation of its describing functions, the following system parameters are defined.

- $\omega_C$ $(=2\pi f_C)$ Resonance frequency of system, given by
$$1/\omega_C^2 = T_C^2 = C_{AT}M_{AC} = C_{MEC}L_{CET}. \tag{6}$$
- $Q_{MC}$ $Q$ of system at $f_C$ considering non-electrical resistances only, given by
$$Q_{MC} = \omega_C C_{MEC} R_{EC}. \tag{7}$$
- $Q_{EC}$ $Q$ of system at $f_C$ considering electrical resistance $R_E$ only, given by
$$Q_{EC} = \omega_C C_{MEC} R_E. \tag{8}$$
- $Q_{TCO}$ Total $Q$ of system at $f_C$ when driven by source resistance of $R_g = 0$, given by
$$Q_{TCO} = Q_{EC}Q_{MC}/(Q_{EC} + Q_{MC}). \tag{9}$$
- $Q_{TC}$ Total $Q$ of system at $f_C$ including all system resistances, given by
$$Q_{TC} = 1/(\omega_C C_{AT} R_{ATC}). \tag{10}$$
- $\alpha$ System compliance ratio, given by
$$\alpha = C_{AS}/C_{AB}. \tag{11}$$

If the system driver is mounted on a baffle which provides the same total air-load mass as the system enclosure, the driver parameters defined in [12, eqs. (12), (13) and (14)] become

$$T_S^2 = 1/\omega_S^2 = C_{AS}M_{AC}, \tag{12}$$

$$Q_{MS} = \omega_S C_{MEC} R_{ES}, \tag{13}$$

$$Q_{ES} = \omega_S C_{MEC} R_E, \tag{14}$$

where $R_{ES} = B^2l^2/S_D^2R_{AS}$ is an electrical resistance representing the driver suspension losses. The driver compliance equivalent volume is unaffected by air-load masses and is in every case [12, eq. (15)]

$$V_{AS} = \rho_o c^2 C_{AS}, \tag{15}$$

where $\rho_o$ is the density of air (1.18 kg/m³) and $c$ is the

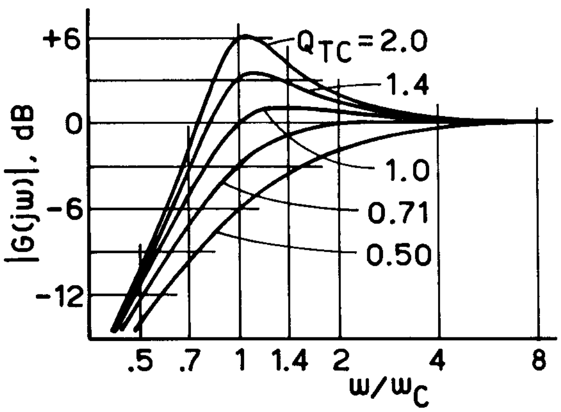


Fig. 4. Normalized amplitude vs normalized frequency response of closed-box loudspeaker system for several values of total system $Q$.

velocity of sound in air (345 m/s). In this paper, the general driver parameters $f_S$ (or $T_S$), $Q_{MS}$ and $Q_{ES}$ will be understood to have the above values unless otherwise specified.

Comparing (1), (6), (8), (11), (12) and (14), the following important relationships between the system and driver parameters are evident:

$$C_{AS}/C_{AT} = \alpha + 1, \tag{16}$$

$$f_C/f_S = T_S/T_C = (\alpha+1)^{1/2}, \tag{17}$$

$$Q_{EC}/Q_{ES} = (\alpha+1)^{1/2}. \tag{18}$$

Following the method of [12], analysis of the circuits of Figs. 2 and 3 and substitution of the parameters defined above yields the system response function

$$G(s) = \frac{s^2 T_C^2}{s^2 T_C^2 + sT_C/Q_{TC} + 1}, \tag{19}$$

the diaphragm displacement function

$$X(s) = \frac{1}{s^2 T_C^2 + sT_C/Q_{TC} + 1}, \tag{20}$$

the displacement constant

$$k_x = 1/(\alpha+1), \tag{21}$$

and the voice-coil impedance function

$$Z_{VC}(s) = R_E + R_{EC}\frac{sT_C/Q_{MC}}{s^2 T_C^2 + sT_C/Q_{MC} + 1}, \tag{22}$$

where $s = \sigma + j\omega$ is the complex frequency variable.

## 3. RESPONSE

### Frequency Response

The response function of the closed-box system is given by (19). This is a second-order (12 dB/octave cutoff) high-pass filter function; it contains information about the low-frequency amplitude, phase, delay and transient response characteristics of the closed-box system [13]. Because the system is minimum-phase, these characteristics are interrelated; adjustment of one determines the others. In audio systems, the flatness and extent of the steady-state amplitude-vs-frequency response—or simply frequency response—is usually considered to be of greatest importance.

The frequency response $|G(j\omega)|$ of the closed-box system is examined in the appendix. Several typical response curves are illustrated in Fig. 4 with the frequency scale normalized to $\omega_C$. The curve for $Q_{TC} = 0.50$ is a second-order critically-damped alignment; that for $Q_{TC} = 0.71$ (i.e., $1/\sqrt{2}$) is a second-order Butterworth ($B2$) maximally-flat alignment. Higher values of $Q_{TC}$ lead to a peak in the response, accompanied by a relative extension of bandwidth which initially is greater than the relative response peak. For large values of $Q_{TC}$, however, the response peak continues to increase without any significant extension of bandwidth. Technically, these responses for $Q_{TC}$ greater than $1/\sqrt{2}$ are second-order Chebyshev ($C2$) equal-ripple alignments.

Whatever response shape may be considered optimum, Fig. 4 indicates the value of $Q_{TC}$ required to achieve this alignment and the variation in response shape that will result if $Q_{TC}$ is altered, i.e., misaligned, from the required value. For intermediate values of $Q_{TC}$ not included in Fig. 4, Fig. 5 gives normalized values of the response peak magnitude $|G(j\omega)|_{max}$, the normalized frequency $f_{G\,max}/f_C$ at which this peak occurs, and the normalized cutoff (half-power) frequency $f_3/f_C$ for which the response is 3 dB below passband level. The analytical expressions for the quantities plotted in Fig. 5 are given in the appendix.

### Transient Response

The response of the closed-box system to a step input is plotted in Fig. 6 for several values of $Q_{TC}$; the time scale is normalized to the periodic time of the system resonance frequency. For values of $Q_{TC}$ greater than 0.50, the response is oscillatory with increasing values of $Q_{TC}$ contributing increasing amplitude and decay time [13].

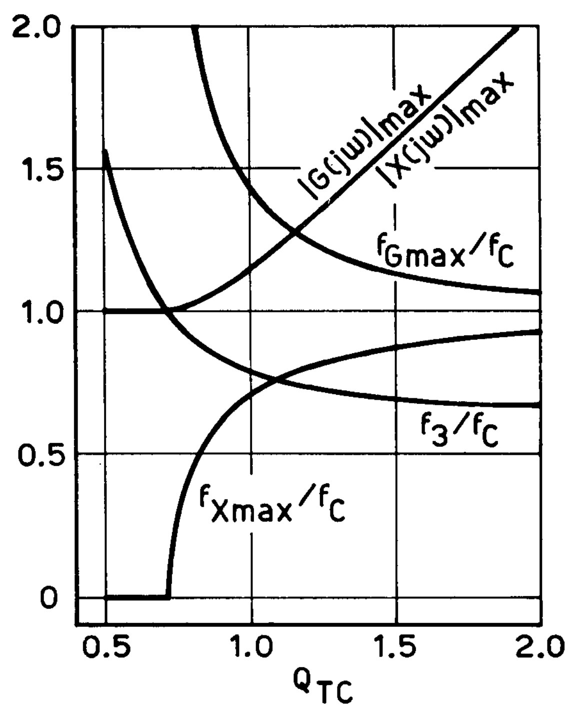


Fig. 5. Normalized cutoff frequency, and normalized frequency and magnitude of response and displacement peaks, as a function of total $Q$ for the closed-box loudspeaker system.

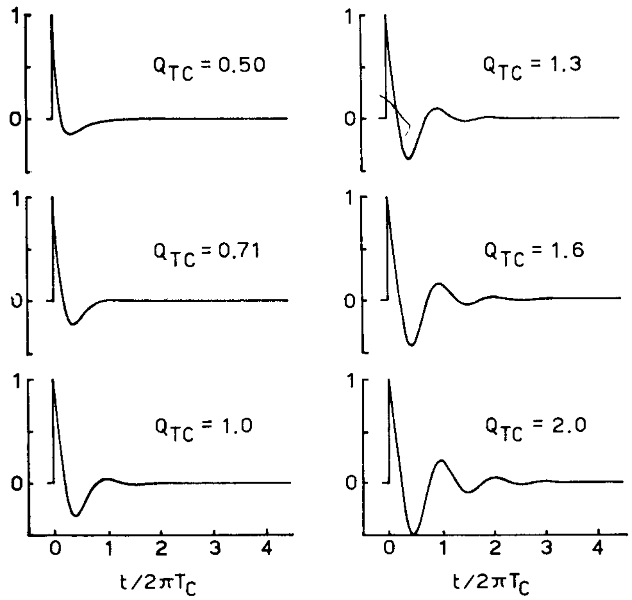


Fig. 6. Normalized step response of the closed-box loudspeaker system.

## 4. EFFICIENCY

### Reference Efficiency

The closed-box system efficiency in the passband region, or system reference efficiency, is the reference efficiency of the driver operating with the particular value of air-load mass provided by the system enclosure. From [12, eq. (32)], this is

$$\eta_o = \frac{4\pi^2}{c^3} \cdot \frac{f_S{}^3 V_{AS}}{Q_{ES}}, \tag{23}$$

where $f_S$, $Q_{ES}$ and $V_{AS}$ have the values given in (12), (14) and (15). This expression may be rewritten in terms of the system parameters defined in section 2. Using (16), (17) and (18),

$$\eta_o = \frac{4\pi^2}{c^3} \cdot \frac{f_C{}^3 V_{AT}}{Q_{EC}}, \tag{24}$$

where

$$V_{AT} = \rho_o c^2 C_{AT} \tag{25}$$

is a volume of air having the same total acoustic compliance as the driver suspension and enclosure acting together. For SI units, the value of $4\pi^2/c^3$ is $9.64 \times 10^{-7}$.

### Efficiency Factors

Equation (24) may be written

$$\eta_o = k_\eta f_3{}^3 V_B, \tag{26}$$

where

- $f_3$ is the cutoff (half-power or $-3$ dB) frequency of the system,
- $V_B$ is the net internal volume of the system enclosure,
- $k_\eta$ is an efficiency constant given by

$$k_\eta = \frac{4\pi^2}{c^3} \cdot \frac{f_C{}^3}{f_3{}^3} \cdot \frac{V_{AT}}{V_B} \cdot \frac{1}{Q_{EC}}. \tag{27}$$

The efficiency constant $k_\eta$ may be separated into three factors: $k_{\eta(Q)}$ related to system losses, $k_{\eta(C)}$ related to system compliances, and $k_{\eta(G)}$ related to the system response. Thus

$$k_\eta = k_{\eta(Q)}\, k_{\eta(C)}\, k_{\eta(G)}, \tag{28}$$

where

$$k_{\eta(Q)} = Q_{TC}/Q_{EC}, \tag{29}$$

$$k_{\eta(C)} = V_{AT}/V_B, \tag{30}$$

$$k_{\eta(G)} = \frac{4\pi^2}{c^3} \cdot \frac{1}{(f_3/f_C)^3 Q_{TC}}. \tag{31}$$

### *Loss Factor*

Modern amplifiers are designed to have a very low output-port (Thevenin) impedance so that, for practical purposes, $R_g = 0$. The value of $Q_{TC}$ for any system used with such an amplifier is then equal to $Q_{TCO}$ as given by (9). Equation (29) then reduces to

$$k_{\eta(Q)} = Q_{TCO}/Q_{EC} = 1 - (Q_{TCO}/Q_{MC}). \tag{32}$$

This expression has a limiting value of unity, but will approach this value only when mechanical losses in the system are negligible ($Q_{MC}$ infinite) and all required damping is therefore provided by electromagnetic coupling ($Q_{EC} = Q_{TCO}$).

The value of $k_{\eta(Q)}$ for typical closed-box systems varies from about 0.5 to 0.9. Low values usually result from the deliberate use of mechanical or acoustical dissipation, either to ensure adequate damping of diaphragm or suspension resonances at higher frequencies, or to conserve magnetic material and therefore cost.

### *Compliance Factor*

Equation (30) may be expanded to

$$k_{\eta(C)} = \frac{C_{AT}}{C_{AB}} \cdot \frac{V_{AB}}{V_B}, \tag{33}$$

where

$$V_{AB} = \rho_o c^2 C_{AB} \tag{34}$$

is a volume of air having an acoustic compliance equal to $C_{AB}$.

There is an important difference between $V_B$, the net internal volume of the enclosure, and $V_{AB}$, a volume of air which represents the acoustic compliance of the enclosure. If the enclosure contains only air under adiabatic conditions, i.e., no lining or filling materials, then $V_{AB}$ is equal to $V_B$. But if the enclosure does contain such materials, $V_{AB}$ is larger than $V_B$. The increase in $V_{AB}$ is inversely proportional to the change in the value of $\gamma$, the ratio of specific heat at constant pressure to that at constant volume for the air in the enclosure. This has a value of 1.4 for the empty enclosure and decreases toward unity if the enclosure is filled with a low-density material of high specific heat [1, p. 220]. Equation (33) may then be simplified to

$$k_{\eta(C)} = \frac{\alpha}{\alpha + 1} \cdot \frac{1.4}{\gamma_B}, \tag{35}$$

where $\gamma_B$ is the value of $\gamma$ applicable to the enclosure.

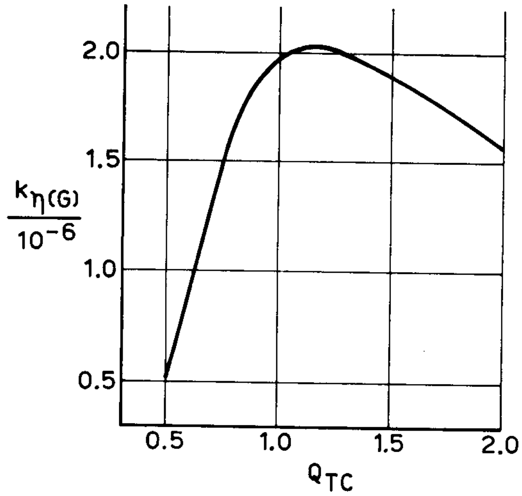


Fig. 7. Response factor $k_{\eta(G)}$ as a function of total $Q$ for the closed-box loudspeaker system.

For "empty" enclosures, (35) has a limiting value of unity for $\alpha >> 1$. Air-suspension systems usually have $\alpha$ values between 3 and 10.

If the enclosure is filled, the $1.4/\gamma_B$ term exceeds unity, but two interactions occur. First, because the filling material increases $C_{AB}$, the value of $\alpha$ is lower than for the empty enclosure. Second, the addition of the material increases energy absorption within the enclosure, decreasing $Q_{MC}$ and therefore reducing the value of $k_{\eta(Q)}$ in (32).

With proper selection of the amount, kind, and location of filling material, the net product of $k_{\eta(Q)}$ and $k_{\eta(C)}$ increases compared to the empty enclosure condition, but the increase is seldom more than about 15%. Haphazard addition of unselected materials may even reduce the product of these factors. Although theoretically possible, it is extremely unusual in practice for this product

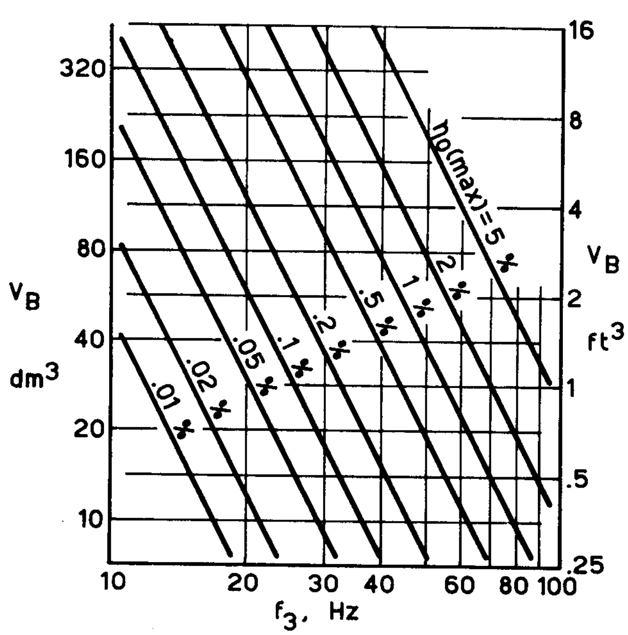


Fig. 8. The relationship of maximum reference efficiency to cutoff frequency and enclosure volume for the closed-box loudspeaker system.

to exceed unity. The effects of filling materials are discussed further in section 7.

### Response Factor

The value of $k_{\eta(G)}$ in (31) depends only on $Q_{TC}$ because $(f_3/f_C)$ is a function of $Q_{TC}$ as shown in Fig. 5 and (75) of the appendix. Fig. 7 is a plot of $k_{\eta(G)}$ *vs* $Q_{TC}$. Just above $Q_{TC} = 1.1$, $k_{\eta(G)}$ has a maximum value of $2.0 \times 10^{-6}$. This value of $Q_{TC}$ corresponds to a $C2$ alignment with a ripple or passband peak of 1.9 dB. Compared to the $B2$ alignment having the same bandwidth, this alignment is 1.8 dB more efficient.

### Maximum Reference Efficiency, Bandwidth, and Enclosure Volume

Selecting the value of $k_{\eta(G)}$ for the maximum-efficiency $C2$ alignment, and taking unity as the maximum attainable value of $k_{\eta(Q)}k_{\eta(C)}$, the maximum reference efficiency $\eta_{o(\max)}$ that could be expected from an idealized closed-box system for specified values of $f_3$ and $V_B$ is, from (26) and (28),

$$\eta_{o(\max)} = 2.0 \times 10^{-6} f_3^3 V_B, \qquad (36)$$

where $f_3$ is in Hz and $V_B$ is in $m^3$. This relationship is illustrated in Fig. 8, with $V_B$ (given here in cubic decimeters—1 $dm^3$ = 1 liter = $10^{-3}$ $m^3$) plotted against $f_3$ for various values of $\eta_{o(\max)}$ expressed in percent.

Figure 8 represents the physical efficiency-bandwidth-volume limitation of closed-box system design. Any system having given values of $f_3$ and $V_B$ must always have an actual reference efficiency lower than the value of $\eta_{o(\max)}$ given by Fig. 8. Similarly, a system of specified efficiency and volume must have a cutoff frequency higher than that indicated by Fig. 8, etc. These basic relationships have been known on a qualitative basis for years (see, e.g., [11]). An independently derived presentation of the important quantitative limitation was given recently by Finegan [14].

There are two known methods of circumventing the physical limitation imposed by (36) or Fig. 8. One is the stabilized negative-spring principle [15] which enables $V_{AT}$ to be made much larger than $V_B$ but requires additional design complexity. The other is the use of amplifier assistance which extends response with the aid of equalizing networks or special feedback techniques [16]. The second method requires additional amplifier power in the region of extended response and a driver capable of dissipating the extra power.

The actual reference efficiency of any practical system may be evaluated directly from (24) if the values of $f_C$, $Q_{EC}$ and $V_{AT}$ are known or are measured. For air-suspension systems, especially those using filling materials, $V_{AT}$ is often very nearly equal to $V_B$.

### Efficiency-Bandwidth-Volume Exchange

The relationship between reference efficiency, bandwidth, and enclosure volume indicated by (26) and illustrated for maximum-efficiency conditions in Fig. 8 implies that these system specifications can be exchanged one for another if the factors determining $k_\eta$ remain constant. Thus if the system is made larger, the parameters may be adjusted to give greater efficiency or extended bandwidth. Similarly, if the cutoff frequency is

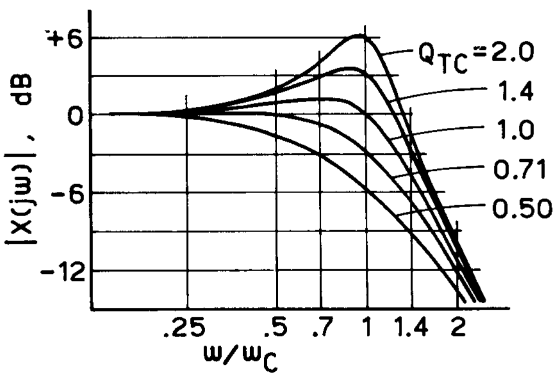


Fig. 9. Normalized diaphragm displacement of closed-box system driver as a function of normalized frequency for several values of total system $Q$.

raised, the parameters may be adjusted to give higher efficiency or a smaller enclosure.

If the value of $k_\eta$ is increased, by reducing mechanical losses, by adding filling material, by increasing $\alpha$, or by changing the response shape, the benefit may be taken in the form of smaller size, or higher efficiency, or extended bandwidth, or a combination of these. Each choice requires a specific adjustment of the enclosure or driver parameters.

## 5. DISPLACEMENT-LIMITED POWER RATINGS

### Displacement Function

The closed-box system displacement function given by (20) is a second-order low-pass filter function. The properties of this function are examined in the appendix.

The normalized diaphragm displacement magnitude $|X(j\omega)|$ is plotted in Fig. 9 with frequency normalized to $\omega_C$ for several values of $Q_{TC}$. The curves are exact mirror images of those of Fig. 4. For intermediate values of $Q_{TC}$, Fig. 5 gives normalized values of the displacement peak magnitude $|X(j\omega)|$ and the normalized frequency $f_{X\max}/f_C$ at which this peak occurs. Analytical expressions for these quantities are given in the appendix.

### Acoustic Power Rating

Assuming linear large-signal diaphragm displacement, the steady-state displacement-limited acoustic power rating $P_{AR}$ of a loudspeaker system, from [12, eq. (42)], is

$$P_{AR} = \frac{4\pi^3\rho_o}{c} \cdot \frac{f_S{}^4 V_D{}^2}{k_x{}^2 |X(j\omega)|_{\max}{}^2}, \quad (37)$$

where $V_D$ is the peak displacement volume of the driver diaphragm, given by

$$V_D = S_D x_{\max}, \quad (38)$$

and $x_{\max}$ is the peak linear displacement of the driver diaphragm, usually set by the amount of voice-coil overhang. Substituting (17) and (21) into (37), the steady-state displacement-limited acoustic power rating of the closed-box system becomes

$$P_{AR(CB)} = \frac{4\pi^3\rho_o}{c} \cdot \frac{f_C{}^4 V_D{}^2}{|X(j\omega)|_{\max}{}^2}. \quad (39)$$

For SI units, the constant $4\pi^3\rho_o/c$ is equal to 0.424.

### Power Output, Bandwidth, and Displacement Volume

Equation (39) may be rewritten as

$$P_{AR(CB)} = k_P f_3{}^4 V_D{}^2, \quad (40)$$

where $k_P$ is a power rating constant given by

$$k_P = \frac{4\pi^3\rho_o}{c} \cdot \frac{1}{(f_3/f_C)^4 |X(j\omega)|_{\max}{}^2}. \quad (41)$$

The acoustic power rating of a system having a specified cutoff frequency $f_3$ and a driver displacement volume $V_D$ is thus a function of $k_P$; and $k_P$ is solely a function of $Q_{TC}$ as shown by (75) and (78) of the appendix.

The variation of $k_P$ with $Q_{TC}$ is plotted in Fig. 10. A maximum value occurs for $Q_{TC}$ very close to 1.1. This is practically the same 1.9 dB ripple $C2$ alignment that gives maximum efficiency. For this condition, (40) becomes

$$P_{AR(CB)\max} = 0.85 f_3{}^4 V_D{}^2, \quad (42)$$

where $P_{AR}$ is in watts for $f_3$ in Hz and $V_D$ in m$^3$.

Equation (42) is illustrated in Fig. 11. $P_{AR}$ is expressed in both watts (left scale) and equivalent SPL at one meter [1, p. 14] for $2\pi$ steradian free-field radiation conditions (right scale); this is plotted as a function of $f_3$ for various values of $V_D$. The SPL at one meter given on the right-hand scale is a rough indication of the level produced in the reverberant field of an average listening room for a radiated acoustic power given by the left-hand scale [1, p. 318].

Figure 11 represents the physical large-signal limitation of closed-box system design. It may be used to determine the optimum performance tradeoffs ($P_{AR}$ *vs* $f_3$) for a given diaphragm and voice-coil design or to find the minimum value of $V_D$ which is required to meet a given specification of $f_3$ and $P_{AR}$. The techniques noted earlier which may be used to overcome the small-signal limitation of Fig. 8 do not affect the large-signal limitation imposed by Fig. 11.

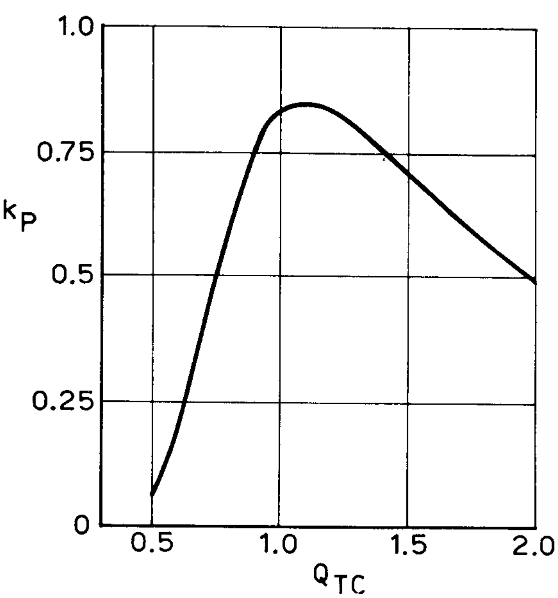


Fig. 10. Power rating constant $k_P$ as a function of total $Q$ for the closed-box loudspeaker system.

### Power Output, Bandwidth, and Enclosure Volume

The displacement-limited power rating relationships given above exhibit no dependence on enclosure volume. For fixed response, it is the diaphragm displacement volume $V_D$ that controls the system power rating. However, $V_D$ cannot normally be made more that a few percent of $V_B$; beyond this point, increases in $V_D$ result in unavoidable non-linear distortion, regardless of driver linearity, caused by non-linear compression of the air in the enclosure [3], [10]. If $V_D$ is limited to a fixed fraction of $V_B$, the fraction depending on the amount of distortion considered acceptable, then Fig. 11 may be relabeled to show the minimum enclosure volume required to provide a given combination of $f_3$ and $P_{AR}$ for the specified distortion level, as well as the required $V_D$.

### Program Bandwidth

Figure 10 indicates that $k_P$ and hence the system steady-state acoustic power rating decreases for values of $Q_{TC}$ below 1.1 if $f_3$ and $V_D$ are held constant. However, it is clear from Fig. 5 that the frequency of maximum diaphragm displacement, $f_{X\max}$, is below $f_3$ for $Q_{TC} < 1.1$, and that as $Q_{TC}$ decreases, $f_{X\max}$ moves further and further below $f_3$. This suggests that the steady-state rating becomes increasingly conservative, as $Q_{TC}$ decreases, for loudspeaker systems operated with program material having little energy content below $f_3$. The effect of restricted power bandwidth in most amplifiers further reduces the likelihood of reaching rated displacement at $f_{X\max}$ for these alignments [12, section 7].

For closed-box loudspeaker systems used for high-fidelity music reproduction and having a cutoff frequency of about 40 Hz or less, or operated on speech only and having a cutoff frequency of about 100 Hz or less, an approximate *program* power rating is that given by (42) or Fig. 11 for any value of $Q_{TC}$ up to 1.1. Above this value, $f_{X\max}$ is within the system passband and the program rating is effectively the same as the steady-state rating.

### Electrical Power Rating

The displacement-limited electrical and acoustic power ratings of a loudspeaker system are related by the system reference efficiency [12, section 7]. Thus, if the acoustic power rating and reference efficiency of a system are known, the corresponding electrical rating may be calculated as the ratio of these.

For the closed-box system, (24) and (39) give the electrical power rating $P_{ER}$ as

$$P_{ER(CB)} = \pi \rho_o c^2 \frac{f_C Q_{EC}}{V_{AT}} \cdot \frac{V_D^2}{|X(j\omega)|_{max}^2}. \qquad (43)$$

The dependence of this rating on the important system constants is more easily observed from the form obtained by dividing (40) by (26):

$$P_{ER} = \frac{k_P}{k_\eta} f_3 \frac{V_D^2}{V_B}. \qquad (44)$$

It is particularly important to realize that for a given acoustic power capacity, the displacement-limited electrical power rating is inversely proportional to efficiency.

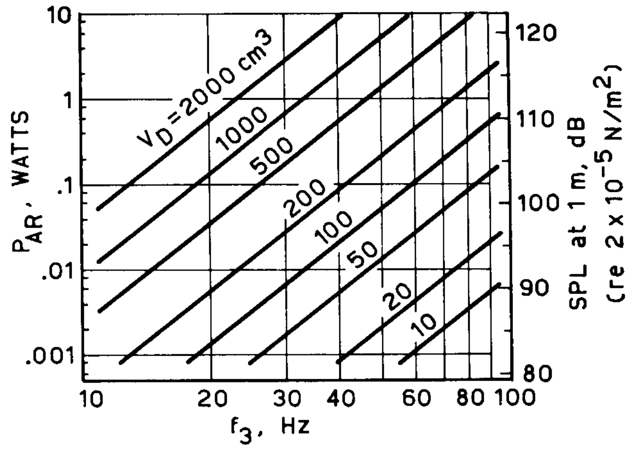


Fig. 11. The relationship of rated acoustic output power to cutoff frequency and driver displacement volume for a closed-box loudspeaker system aligned to obtain maximum rated power.

Also, displacement non-linearity for large signals tends to increase $P_{ER}$ over the theoretical linear value. Thus a high input power rating is not necessarily a virtue; it may only indicate a low value of $k_\eta$ or a high distortion limit.

The overall electrical power rating which a manufacturer assigns to a loudspeaker system must take into account both the displacement-limited power capacity of the system, $P_{ER}$, and the thermally-limited power capacity of the driver, $P_{E(max)}$, together with the spectral and statistical properties of the type of program material for which the rating will apply. The statistical properties of the signal are important in determining whether $P_{ER}$ or $P_{E(max)}$ will limit the overall power rating, because the overall rating sets the maximum safe continuous-power rating of the amplifier to be used. For reliability and low distortion, the overall rating must never exceed $P_{ER}$; but it may be allowed to exceed $P_{E(max)}$ in proportion to the peak-to-average power ratio of the intended program material.

The resulting system rating is important when selecting a loudspeaker system to operate with a given amplifier and vice-versa. But it must be remembered that the electrical rating gives no clue to the acoustic power capacity unless the reference efficiency is known.

## 6. PARAMETER MEASUREMENT

It has been shown that the important small-signal and large-signal performance characteristics of a closed-box loudspeaker system depend on a few basic parameters. The ability to measure these basic parameters is thus a useful tool, both for evaluating the performance of an existing loudspeaker system and for checking the results of a new system design which is intended to meet specific performance criteria.

### Small-Signal Parameters: $f_C$, $Q_{MC}$, $Q_{EC}$, $Q_{TCO}$, $\alpha$, $V_{AT}$

The voice-coil impedance function of the closed-box system is given by (22). The steady-state magnitude $|Z_{VC}(j\omega)|$ of this function is plotted against normalized frequency in Fig. 12.

The measured impedance curve of a closed-box sys-

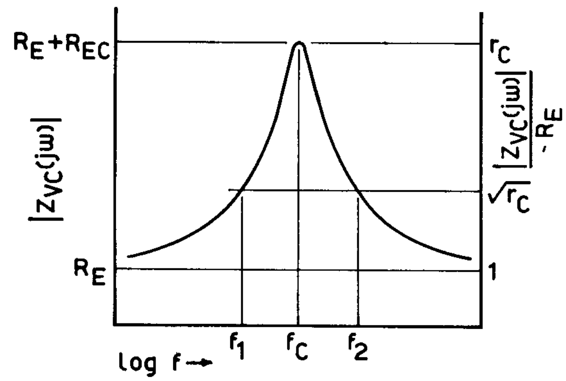


Fig. 12. Magnitude of closed-box loudspeaker system voice-coil impedance as a function of frequency.

tem conforms closely to the shape of Fig. 12. This impedance curve permits identification of the first four parameters as follows:

1) Measure the dc voice-coil resistance $R_E$.
2) Find the frequency $f_C$ at which the impedance has maximum magnitude and zero phase, i.e., is resistive. Let the ratio of maximum impedance magnitude to $R_E$ be defined as $r_C$.
3) Find the two frequencies $f_1 < f_C$ and $f_2 > f_C$ for which the impedance magnitude is equal to $R_E\sqrt{r_C}$.
4) Then, as in [12, appendix],

$$Q_{MC} = \frac{f_C\sqrt{r_C}}{f_2 - f_1}, \quad (45)$$

$$Q_{EC} = Q_{MC}/(r_C - 1), \quad (46)$$

$$Q_{TCO} = Q_{MC}/r_C. \quad (47)$$

To obtain the value of $\alpha$ for the system, remove the driver from the enclosure and measure the driver parameters $f_S$, $Q_{MS}$ and $Q_{ES}$ (with or without a baffle) as described in [12]; the method is the same as that given above for the system. The compliance ratio is then [12, appendix]

$$\alpha = \frac{f_C Q_{EC}}{f_S Q_{ES}} - 1. \quad (48)$$

Drivers with large voice-coil inductance or systems having a large crossover inductance may exhibit some difference between the frequency of maximum impedance magnitude and the frequency of zero phase. If the inductance cannot be bypassed or equalized for measurement purposes [17, section 14], it is better to take $f_C$ as the frequency of maximum impedance magnitude, regardless of phase. It must be expected, however, that some measurement accuracy will be lost in these circumstances.

$V_{AT}$ is evaluated with the help of (1), (11), (15), (25) and (34):

$$V_{AT} = V_{AB}V_{AS}/(V_{AB} + V_{AS}) = \frac{\alpha}{\alpha + 1} V_{AB}. \quad (49)$$

For unfilled enclosures, $V_{AB} = V_B$ and the value of $V_{AT}$ may be computed directly using the measured value of $\alpha$. If the system enclosure is normally filled, an extra set of measurements is required. The filling material is removed from the enclosure, or the driver is transferred to a similar but unfilled test enclosure. For this combination, the resonance frequency $f_{CT}$ and the corresponding $Q$ values $Q_{MCT}$ and $Q_{ECT}$ are measured by the above method. Then, as shown in [12, appendix],

$$V_{AS} = V_B\left[\frac{f_{CT}Q_{ECT}}{f_S Q_{ES}} - 1\right], \quad (50)$$

where $V_B$ is the net internal volume of the unfilled enclosure used ( the system enclosure or test enclosure). Using (11), (15) and (34), $V_{AB}$ for the filled system enclosure is then given by

$$V_{AB} = V_{AS}/\alpha. \quad (51)$$

This value of $V_{AB}$ may now be used to evaluate $V_{AT}$ using (49).

### Large-Signal Parameters: $P_{E(max)}$ and $V_D$

The measurement of driver thermal power capacity is best left to manufacturers, who are familiar with the required techniques [18, section 5.7] and are usually quite happy to supply the information on request. Some estimate of thermal power capacity may often be obtained from knowledge of voice-coil diameter and length, the materials used, and the intended use of the driver [19].

The driver displacement volume $V_D$ is the product of $S_D$ and $x_{max}$. It is usually sufficient to evaluate $S_D$ by estimating the effective diaphragm diameter. Some manufacturers specify the "throw" of a driver, which is usually the peak-to-peak linear displacement, i.e., $2x_{max}$. If this information is not available, the value of $x_{max}$ may be estimated by observing the amount of voice-coil overhang outside the magnetic gap. For a more rigorous evaluation, where the necessary test equipment is available, operate the driver in air with sine-wave input at its resonance frequency and measure the peak displacement for which the radiated sound pressure attains about 10% total harmonic distortion.

## 7. ENCLOSURE FILLING

It is stated in section 4 that the addition of an appropriate filling material to the enclosure of an air-suspension system raises the value of the efficiency constant $k_\eta$. The use and value of such materials have been the subject of much controversy and study [4], [8], [9], [10], [11], [20].

There is no serious disagreement about the value of such materials for damping standing waves within the enclosure at frequencies in the upper piston range and higher. The controversy centers on the value of the materials at low frequencies. A more complete description of the effects of these materials will help to assess their value to various users.

### Compliance Increase

If the filling material is chosen for low density but high specific heat, the conditions of air compression within the enclosure are altered from adiabatic to isothermal, or partly so [1, p. 220]. This increases the effective acoustic compliance of the enclosure, which is equivalent to increasing the size of the unfilled enclosure. The maximum theoretical increase in compliance is 40%, but using practical materials the actual increase is probably never more than about 25%.

## Mass Loading

Often, the addition of filling material increases the total effective moving mass of the system. This has been carefully documented by Avedon [10]. The mechanism is not entirely clear and may involve either motion of the filling material itself or constriction of air passages near the rear of the diaphragm, thus "mass-loading" the driver. Depending on the initial diaphragm mass and the conditions of filling, the mass increase may vary from negligible proportions to as much as 20%.

## Damping

Air moving inside a filled enclosure encounters frictional resistance and loses energy. Thus the component $R_{AB}$ of Fig. 1 increases when the enclosure is filled. The resulting increase in the total system mechanical losses $(R_{AB} + R_{AS})$ can be substantial, especially if the filling material is relatively dense and is allowed to be quite close to the driver where the air particle velocity and displacement are highest. While unfilled systems have typical $Q_{MC}$ values of about 5–10 (largely the result of driver suspension losses), filled systems generally have $Q_{MC}$ values in the range of 2–5.

## Value to the Designer

If a loudspeaker system is being designed from scratch, the effect of filling material on compliance is a definite advantage. It means that the enclosure size can be reduced or the efficiency improved or the response extended. Any mass increase which accompanies the compliance increase is simply taken into account in designing the driver so that the total moving mass is just the amount desired. The losses contributed by the material are a disadvantage in terms of their effect on $k_{\eta(Q)}$, but this is a small price to pay for the overall increase in $k_\eta$ which results from the greater compliance. In fact, if efficiency is not a problem, the effect of increased frictional losses may be seen to relax the magnet requirements a little, thus saving cost.

Where a loudspeaker system is being designed around a given driver, the compliance increase contributed by the material is still an advantage because it permits the enclosure to be made smaller for a particular (achievable) response. The effect of increased mass is to reduce the driver reference efficiency by the square of the mass increase; this may or may not be desirable. The increased mass will also cause the value of $Q_{EC}$ to be higher for a given value of $f_C$. This will be opposed by the effect of the material losses on $Q_{MC}$.

Often it is hoped that the addition of large amounts of filling material to a system will contribute enough additional damping to compensate for inadequate magnetic coupling in the driver. To the extent that the material increases compliance more than it does mass, $Q_{EC}$ will indeed fall a little. And while $Q_{MC}$ may be substantially decreased, the total reduction in $Q_{TC}$ is seldom enough to rescue a badly underdamped driver as illustrated in [20]. If such a driver must be used, the application of acoustic damping directly to the driver as described in [21] is both more effective and more economical than attempting to overfill the enclosure.

## Measuring the Effects of Filling Materials

The contribution of filling materials to a given system can be determined by careful measurement of the system parameters with and without the material in place. The added-weight measurement method used by Avedon [10] can be very accurate but is suited only to laboratory conditions. Alternatively, the type of measurements described in section 6 may be used:

1) With the driver in air or on a test baffle, measure $f_S$, $Q_{MS}$, $Q_{ES}$.
2) With the driver in the unfilled enclosure, measure $f_{CT}$, $Q_{MCT}$, $Q_{ECT}$.
3) With the driver in the filled enclosure, measure $f_C$, $Q_{MC}$, $Q_{EC}$.
4) Then, using the method of [12, appendix], the ratio of total moving mass with filling to that without filling is

$$M_{AC}/M_{ACT} = f_{CT}Q_{EC}/f_C Q_{ECT}, \tag{52}$$

and the enclosure compliance increase caused by filling is

$$V_{AB}/V_B = \frac{(f_{CT}Q_{ECT}/f_S Q_{ES}) - 1}{(f_C Q_{EC}/f_S Q_{ES}) - 1}. \tag{53}$$

5) The net effect of the material on total system damping may be found by computing $Q_{TCO}$ for the filled system from (9) or (47) and comparing this to the corresponding $Q_{TCTO} = Q_{MCT}Q_{ECT}/(Q_{MCT} + Q_{ECT})$ for the unfilled system. These values represent the total $Q$ $(Q_{TC})$ for each system when driven by an amplifier of negligible source resistance.

The usual result is that the filling material increases both compliance and mass but decreases total $Q$. The decrease in total $Q$ may be a little or a lot, depending on the initial value and on the material chosen and its location in the enclosure.

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## THE AUTHOR

Richard H. Small was born in San Diego, California in 1935. He received the degrees of Bachelor of Science (1956) from the California Institute of Technology and Master of Science in Electrical Engineering (1958) from the Massachusetts Institute of Technology.

He was employed in electronic circuit design for high-performance analytical instruments at the Bell & Howell Research Center from 1958 to 1964, except for a one-year visiting fellowship to the Norwegian Technical University in 1962. After a working visit to Japan in 1964, he moved to Australia where he has been associated with the School of Electrical Engineering of The University of Sydney. In 1972 he was awarded the degree of Doctor of Philosophy following the completion of a program of research into direct-radiator electrodynamic loudspeaker systems.

Dr. Small is a member of the Audio Engineering Society, the Institute of Electrical and Electronics Engineers, and the Institution of Radio and Electronics Engineers, Australia. He is also a member of the Subcommittee on Loudspeaker Standards of the Standards Association of Australia.

# Closed-Box Loudspeaker Systems Part II: Synthesis

**RICHARD H. SMALL**

*School of Electrical Engineering, The University of Sydney*
*Sydney, N.S.W. 2006, Australia*

Part I of this paper provides a basic low-frequency analysis of the closed-box loudspeaker system with emphasis on small-signal and large-signal behavior, basic performance limitations, and the determination of important system parameters from voice-coil impedance measurements. Part II discusses some important implications of the findings of Part I and introduces the subject of system synthesis: the complete design of loudspeaker systems to meet specific performance goals. Given a set of physically-realizable system performance specifications, the analytical results of Part I enable the system designer to calculate directly the required specifications of the system components.

**Editor's Note:** Part I of Closed-Box Loudspeaker Systems appeared in the December 1972 issue of the Journal.

## 8. DISCUSSION

### Driver Size

It has long been an accepted principle that a large bass driver is better than a small one. While this attitude seems to be justified by experience, it has recently been called into question [22]. The analysis in this paper demonstrates that driver size alone does not determine or limit system performance in areas of small-signal response, efficiency, or displacement-limited power capacity.

A large driver inevitably costs more than a small driver having identical small-signal and large-signal parameters of the kind discussed here. However, it is physically easier to obtain a large value of $V_D$ and hence a high acoustic power capacity from a large driver, and the modulation distortion [23] produced by a large driver will be less than that of a small driver delivering the same acoustic output power.

Thus a large driver has no inherent advantage over a small one so far as small-signal response and efficiency are concerned. It may in fact have a cost disadvantage. But where high acoustic output at low distortion is required, the large driver has a definite advantage.

### Enclosure Size

It is clear from section 4 that an air-suspension system having a high compliance ratio can duplicate the performance of a larger conventional closed-box system having a low compliance ratio. However, once the compliance ratio is made larger than about 4, there is no way to gain a significant reduction in enclosure size without affecting system performance.

A small air-suspension system, when compared to a large air-suspension system, must have a higher cutoff frequency, or lower efficiency, or both. As has been claimed many times, it is possible to design a small system to have the same *response* as a large system. But if both are non-wasteful air-suspension designs, then as shown by (26) or Fig. 8 the efficiency of the small system must be lower than that of the large system in direct proportion to size.

It is often possible to provide the same maximum acoustic output as well as the same response from the small system, but the lower efficiency of this system will dictate a higher input power rating and therefore a driver voice coil capable of dissipating more heat. Also, it is easily shown that for these conditions the driver of the small system will require a larger magnet (e.g., a heavier diaphragm of the same size may be driven through the same displacement, or a smaller diaphragm of the same mass may be driven through a larger displacement). Thus for this condition the driver for the small system must be more expensive than that for the large system.

It may be concluded that the pressure to design more and more compact high-quality loudspeaker systems leads directly to systems of reduced efficiency and, in most cases, reduced acoustic power capacity. If acoustic power capacity is not sacrificed, these compact systems require expensive drivers and must be used with powerful amplifiers.

### Performance Specifications

Of all the components used in audio recording and reproduction, loudspeaker systems have the least complete and least informative performance specifications. In the low-frequency range at least, this need not be so.

If a specified voltage is applied to a direct-radiator loudspeaker system, the output of the system at low frequencies may be expressed in terms of an acoustic volume velocity which is *substantially independent of the acoustic load* [12], [24]. The "response" of a loudspeaker system expressed in this way is meaningless to most loudspeaker users, but a specification of the acoustic power or distant sound pressure delivered into a standard free-field load by this volume velocity is both meaningful and useful.

While the sound pressure delivered to a room is different from that delivered to a free field, the difference clearly is a property of the room, not of the loudspeaker system. If the room performance is very poor, it can be corrected acoustically or, in some cases, equalized electronically. This is in no way a deterrent to accurate specification of the basic loudspeaker system response by using a standard free-field load. In fact, the findings of Allison and Berkovitz [25] indicate that a $2\pi$ sr free-field load is a very reasonable approximation to a typical room load.

Such a standard-load approach has of course been used for years in loudspeaker measurement standards [18], [26], [27]. If it were applied more universally, it would provide a very useful—and presently unavailable—quantitative means of comparing loudspeaker systems. It is a particularly attractive method for specifying the low-frequency response of a system, because the nominal free-field low-frequency response and reference efficiency can be obtained quite easily from the basic parameters of the system.

A few manufacturers already supply these basic parameters or the directly-related free-field response and efficiency data. The practice deserves encouragement.

### Typical System Performance

A sampling of closed-box systems of British, American and European origin was tested in late 1969 by measuring the system small-signal parameters as described in section 6. The frequency response and efficiency were then obtained from the relationships of sections 3 and 4.

Resonance frequencies ($f_C$) varied from 40 Hz to 90 Hz. Total $Q$ ($Q_{TCO}$) varied from 0.4 to 2.0. Reference efficiencies ($\eta_o$) varied from 0.28% to 1.0%. While there was no general pattern of parameter combinations, all but a few of the systems fell into one of two categories:

1) Cutoff frequency ($f_3$) below 50 Hz with little or no peaking ($Q_{TCO}$ up to 1.1). Size generally larger than 40 dm$^3$ (1.4 ft$^3$).
2) Cutoff frequency above 50 Hz with definite peaking ($Q_{TCO}$ between 1.4 and 2.0). Size smaller than 60 dm$^3$ (2 ft$^3$)

One explanation for this situation was spontaneously provided (and demonstrated) by a salesman who sold American systems in both categories. Only category 1 systems would reproduce low organ and orchestral fundamentals, while category 2 systems had demonstrably stronger bass on popular music. Sales thus tended to be determined by the musical tastes of the customer. There is marketing sense in this, and economic sense as well, because the same driver which has category 1 performance in a large enclosure has category 2 performance—with a higher acoustic power capacity—in a small enclosure.

## 9. SYSTEM SYNTHESIS

### System-Driver Relationships

The majority of closed-box systems operate with amplifiers having negligible output resistance, have a total moving mass no greater than that of the driver on a baffle, and obtain most of their total damping from electromagnetic coupling and mechanical losses in the driver. For these conditions, (7), (9), (13), (17) and (18) may be used to derive

$$\frac{Q_{TCO}}{Q_{TS}} \approx \frac{Q_{EC}}{Q_{ES}} = \frac{f_C}{f_S} = (\alpha + 1)^{1/2}, \qquad (54)$$

and thus

$$f_C/Q_{TCO} \approx f_S/Q_{TS}, \qquad (55)$$

where $Q_{TS}$ is the total $Q$ of the driver at $f_S$ for zero source resistance [12, eq. (47)], i.e.,

$$Q_{TS} = Q_{ES}Q_{MS}/(Q_{ES} + Q_{MS}). \qquad (56)$$

These equations show that for any enclosure-driver combination (i.e., value of $\alpha$) the system resonance frequency and $Q$ will be in the same ratio as those of the driver, but individually raised by a factor $(\alpha + 1)^{1/2}$. This increase is plotted as a function of $\alpha$ in Fig. 13.

This approximate relationship and the basic response, efficiency and power capacity relationships derived earlier are used below to develop system design procedures for two important cases: that of a fixed driver design, and that of only the final system specifications given.

## Design with a Given Driver

One difficulty of trying to design an enclosure to "fit" a given driver is that the driver may be completely unsuitable in the first place. A convenient test of suitability for closed-box system drivers is provided by (51) and (54); the driver parameters must be known, or measured.

Equation (54) insists that the driver resonance frequency must always be lower than that of the system. If the designer wishes to avoid an enclosure which is wastefully large, i.e., he desires an air-suspension system, then $\alpha$ must be at least 3 and the driver resonance frequency must be no more than half the maximum tolerable system resonance frequency.

Similarly, $Q_{TS}$ must be lower than the highest acceptable value of $Q_{TCO}$, and by approximately the same factor which relates $f_S$ to the desired or highest acceptable value of $f_C$.

Finally, from (51), the value of $V_{AS}$ must be at least several times larger than the enclosure size desired.

If the driver parameters appear satisfactory, the design of the system is carried out by selecting the most desirable combination of $f_C$ and $Q_{TCO}$ which satisfies (55) and then calculating $\alpha$ from (17). The required enclosure size (net internal volume) is then, from (51),

$$V_B = V_{AS}/\alpha, \tag{57}$$

or somewhat smaller if the enclosure is filled.

The reference efficiency is calculated from (23), and the acoustic power rating from (39) or (42). The electrical power rating is then, from section 5,

$$P_{ER} = P_{AR}/\eta_o. \tag{58}$$

## Example of Design with a Given Driver

Using a standard baffle and unlined test enclosure, a European-made 12-inch woofer sold for air-suspension use is found to have the following small-signal parameters:

$f_S = 19$ Hz
$Q_{MS} = 3.7$
$Q_{ES} = 0.35$
$V_{AS} = 540$ dm$^3$ (19 ft$^3$).

Using (56) and (23),

$Q_{TS} = 0.32$
$\eta_o = 1.02\%$.

The manufacturer's power rating is 25 W, and the peak linear displacement is estimated to be 6 mm (¼ in). The effective diaphragm radius is estimated to be 0.12 m, giving $S_D = 4.5 \times 10^{-2}$ m$^2$ and $V_D = 2.7 \times 10^{-4}$ m$^3$ or 270 cm$^3$.

The values of $f_S$, $Q_{TS}$ and $V_{AS}$ for this driver appear to be quite favorable. The values of $f_C$, $Q_{TCO}$ and $f_3$ to be expected from various suitable values of $\alpha$ are given in Table 1 together with the corresponding enclosure compliance $V_{AB}$ (volume of an unfilled enclosure).

The $\alpha = 4$ alignment gives almost exactly a $B2$ response

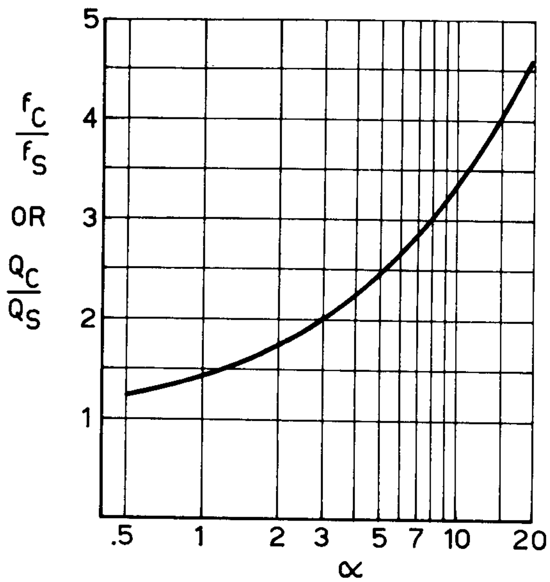


Fig. 13. Ratio of closed-box system resonance frequency and $Q$ to driver resonance frequency and $Q$ as a function of the system compliance ratio $\alpha$.

for an unfilled enclosure volume of 135 dm$^3$ or 4.8 ft$^3$. This would be quite suitable for a floor-standing system. The $\alpha = 9$ alignment gives excellent performance in a volume of only 60 dm$^3$ (2.1 ft$^3$). The $\alpha = 12$ alignment could probably be achieved in a 40 dm$^3$ (1.4 ft$^3$) enclosure with filling. $Q_{TCO}$ would then be lower than shown, probably about unity, giving a cutoff frequency of about 53 Hz. This would be quite adequate "bookshelf" performance.

Taking the larger $B2$-aligned system, the displacement-limited acoustic power rating for program material, from (42), is

$$P_{AR} = 0.19 \text{ W},$$

and the corresponding electrical power rating is

$$P_{ER} = 19 \text{ W}.$$

This is well within the power rating given by the manufacturer, so the system can safely be operated with an amplifier having a continuous power rating of 20 W.

The "bookshelf" design, because of its higher value of $f_3$, has displacement-limited ratings of about 0.5 W acoustical and 50 W electrical. This is much higher than the manufacturer's rating. In the absence of the actual value of $P_{E(max)}$ on which the manufacturer's rating is based, it is probably best to limit the amplifier power to 25 W. The system can then produce an acoustic output of 0.25 W.

## Design from Specifications

Most engineering products are designed to suit specific requirements. Quite commonly, the "requirements" for a particular product contain conflicting factors, and the

Table 1. Expected Performance of the Given Driver

| $\alpha$ | $f_C$, Hz | $Q_{TCO}$ | $f_3$, Hz | $V_{AB}$, dm$^3$ |
|---|---|---|---|---|
| 4 | 42.5 | 0.72 | 42 | 135 |
| 6 | 50.3 | 0.85 | 44 | 90 |
| 9 | 60.0 | 1.01 | 47 | 60 |
| 12 | 68.6 | 1.15 | 50 | 45 |

engineer is called upon to assess the requirements and to adjust them to a condition of physical and economic realizability. Fig. 8, for example, frustrates the desires of many marketing managers who would be delighted to offer a one cubic foot (28 dm$^3$) air-suspension system giving flat response to 20 Hz at high efficiency.

The desired response of a closed-box loudspeaker system may be based on amplitude, phase, delay or transient considerations [13], but can always be reduced to a specification of $f_C$ and $Q_{TC}$. Once the response is specified, either the enclosure volume $V_B$ or the reference efficiency $\eta_o$ may be specified independently; the other will then be determined or restricted to a minimum or maximum value. Finally, the power capacity may be specified in terms of either $P_{ER}$ or $P_{AR}$. If both $P_{ER}$ and $P_{AR}$ must be fixed independently, this will detemine $\eta_o$ and thus restrict $V_B$ as above.

A typical set of design specifications might start with values of $f_C$, $Q_{TC}$, $V_B$ and $P_{AR}$, together with a rating impedance which fixes $R_E$. Unless a special amplifier is to be used, it can be assumed that $Q_{TC} = Q_{TCO}$. Note that $V_B$ effectively specifies the enclosure; the design problem is then to specify the driver.

The design process begins by assigning realistic values to $Q_{MC}$ and $\alpha$. The value of $Q_{MC}$ has only a relatively minor effect on system performance through $k_{\eta(Q)}$. As noted in section 7, typical values are 2–5 for systems using filling material and 5–10 for unfilled systems. If no better guide to the expected value of $Q_{MC}$ is available, assume $Q_{MC} = 5$. The required value of $Q_{EC}$ for the system is then calculated from (9).

If maximum efficiency consistent with the initial specifications is desired, then the air-suspension principle must be used. This requires that $\alpha$ be at least 3 or 4, but its value will otherwise have only a small effect on system performance through $k_{\eta(C)}$ and may be chosen to have any value consistent with physical realizability of the driver. If $\alpha$ is chosen too large, the driver will be found to require unrealistically high compliance which, if realizable at all, may lead to poor mechanical stability of the suspension. A suitable choice of $\alpha$ is usually in the range of 3–10.

Next, the value of $V_{AB}$ is established. This is equal to $V_B$ for unfilled systems, but is increased by the factor $1.4/\gamma_B$ (typically 1.15 to 1.2) if the enclosure is filled.

The required driver small-signal parameters are then, from (17) and (18),

$$f_S = f_C/(\alpha + 1)^{1/2}, \tag{59}$$

$$Q_{ES} = Q_{EC}/(\alpha + 1)^{1/2}, \tag{60}$$

and

$$V_{AS} = \alpha V_{AB}. \tag{51}$$

$V_{AT}$ is determined from (49). The reference efficiency to be expected from the completed system is calculated from (24). Alternatively, $k_{\eta(Q)}$, $k_{\eta(C)}$ and $k_{\eta(G)}$ may be evaluated separately and $\eta_o$ determined from (26). The system electrical power rating $P_{ER}$ is then calculated from (58). A comparable or lower value is assigned to $P_{E(\max)}$, depending on the peak-to-average power ratio of the program material with which the system will be used.

The required value of $V_D$ is calculated directly from (39) using Fig. 5 or (78) to determine $|X(j\omega)|_{\max}$, or from (42), as appropriate. This value must be no larger than a few percent of $V_B$.

The driver is now specified by its most important parameters $f_S$, $Q_{ES}$, $V_{AS}$, $V_D$ and $P_{E(\max)}$ as well as its voice-coil resistance $R_E$ which is typically 80% of the desired rating impedance. The system designer is faced with the problem of obtaining a driver which has the required parameters. If he has a driver factory available, he may have the required driver fabricated as described in the next section. If he does not possess this luxury, he must find a driver from among those available on the market.

At present, very few of the loudspeaker drivers offered for sale are provided with complete parameter information, either in the form above or any other. While this situation will no doubt improve with time, particularly as increasing demands are made on manufacturers to provide such information, today's system designer must obtain samples where possible and measure the parameters as described in [12]. The small-signal parameters should be measured with the driver mounted on a standard test baffle having an area of one or two square meters, e.g., [18, section 4.4.1], so that the diaphragm air load is approximately that which will apply to the driver in the system enclosure.

## Example of System Design from Specifications

A closed-box air-suspension loudspeaker system to be used with a high-damping-factor amplifier is to be designed to meet the following specifications:

| | |
|---|---|
| $f_3$ | 40 Hz |
| Response | $B2$ |
| $V_B$ | 2 ft$^3$ (56.6 dm$^3$) |
| $P_{AR}$ | 0.25 W program peaks; expected peak/average ratio 5 dB. |

The enclosure is to be lined, but not filled. It is assumed that the enclosure and driver losses will correspond to $Q_{MC} = 5$ and that it will be physically possible to obtain a compliance ratio of $\alpha = 5$.

The first two specifications translate directly into

$$f_C = 40\ \text{Hz}$$

and

$$Q_{TC} = Q_{TCO} = 0.707.$$

For $Q_{MC} = 5$, (9) gives

$$Q_{EC} = 0.824.$$

For $\alpha = 5$, $(\alpha + 1)^{1/2} = \sqrt{6} = 2.45$, so from (59) and (60),

$$f_S = 16.3\ \text{Hz}$$

and

$$Q_{ES} = 0.336.$$

Also, for the unfilled enclosure, (51) gives

$$V_{AS} = 10\ \text{ft}^3\ (283\ \text{dm}^3).$$

Then, from (49),

$$V_{AT} = 1.67\ \text{ft}^3\ (47.2\ \text{dm}^3).$$

From (29), (30) and (31),

$$k_{\eta(Q)} = 0.858,$$
$$k_{\eta(C)} = 0.833,$$
$$k_{\eta(G)} = 1.36 \times 10^{-6}.$$

Thus

$$k_\eta = 0.97 \times 10^{-6}$$

and from (26),

$$\eta_o = 0.00351 \text{ or } 0.35\%.$$

The reference efficiency can also be calculated directly from (24) because $f_C$, $V_{AT}$ and $Q_{EC}$ are known.

The displacement-limited electrical power rating, from (58), is

$$P_{ER} = 71.5 \text{ W}.$$

An amplifier of this power rating must be used to obtain the specified acoustic output. For the expected peak/average power ratio, the thermal rating $P_{E(max)}$ of the driver must be at least 22.5 W.

Using (42) for the program power rating,

$$V_D = 3.4 \times 10^{-4} \text{ m}^3 \text{ or } 340 \text{ cm}^3.$$

This is only 0.6% of $V_B$, so linearity of the air compliance is no problem.

## 10. DRIVER DESIGN

### General Method

The process of system design leads to specification of the required driver in terms of basic parameters. These parameters are used to carry out the physical design of the driver.

First, $V_D$ must be divided into acceptable values of $S_D$ and $x_{max}$. The choice of $S_D$ may have to be a compromise among cost, distortion, and available mounting area.

The required mechanical compliance of the diaphragm suspension is then

$$C_{MS} = C_{AS}/S_D^2 = V_{AS}/(\rho_o c^2 S_D^2), \qquad (61)$$

and the required total mechanical moving mass is

$$M_{MS} = 1/[(2\pi f_S)^2 C_{MS}]. \qquad (62)$$

This total moving mass includes any mass added by filling material, as well as the air loads $M_{M1}$ and $M_{MB}$ on front and rear of the diaphragm. The latter can be evaluated from [1, pp. 216-217]. The mechanical mass of the diaphragm and voice-coil assembly is then

$$M_{MD} = M_{MS} - (M_{M1} + M_{MB}), \qquad (63)$$

less any allowance for mass added by filling material.

The magnet and voice coil must provide electromagnetic damping given by

$$B^2l^2/R_E = 2\pi f_S M_{MS}/Q_{ES}, \qquad (64)$$

or, for the value of $R_E$ specified, a $Bl$ product given by

$$Bl = (2\pi f_S R_E M_{MS}/Q_{ES})^{1/2}. \qquad (65)$$

This $Bl$ product, together with the mechanical compliance, must be maintained with good linearity for a diaphragm displacement of $\pm x_{max}$. This effectively means that the voice-coil overhang outside the gap must be about $x_{max}$ at each end. Also, the voice coil must be capable of dissipating as heat, without damage, an electrical input power $P_{E(max)}$. This design problem is familiar to driver manufacturers.

The driver parameter $Q_{MS}$ usually plays a minor role in system performance, but it cannot be neglected entirely. The value of $Q_{MS}$ in practical designs is often affected by decisions related to performance at higher frequencies. Where the driver diaphragm is required to be free of strong resonance modes at high frequencies, the outer edge suspension is usually designed to reflect a minimum of the vibrational energy travelling outward from the voice coil through the diaphragm material. This means that energy is dissipated in the suspension, and a low value of $Q_{MS}$ results. The intended use of the driver or the constructional methods preferred by the manufacturer thus determines the approximate value of $Q_{MS}$. In a completed closed-box system, the value of $Q_{MS}$ and the enclosure and filling material losses determine $Q_{MC}$ and therefore the value of $k_{\eta(Q)}$ for the system.

### Drivers for Air-Suspension Systems

It was stated earlier that the compliance ratio of an air-suspension system is not very important so long as it is greater than about 3 or 4. This means that the exact values of driver compliance, resonance frequency and $Q$ are not of critical importance. It is in fact the moving mass $M_{MS}$ and the electromagnetic damping $B^2l^2/R_E$ that are of greatest importance. These can be calculated directly from the system parameters alone. Substituting (16), (17) and (18) into (61), (62) and (64), or using (3), (6), (8) and (25),

$$M_{MS} = S_D^2 M_{AC} = \rho_o c^2 S_D^2/(4\pi^2 f_C^2 V_{AT}), \qquad (66)$$

and

$$B^2l^2/R_E = 2\pi f_C M_{MS}/Q_{EC}. \qquad (67)$$

The exact value of mechanical compliance is not critically important so long as it is high enough to give approximately the desired compliance ratio. This is an advantage of the air-suspension design principle, because mechanical compliance is one of the more difficult driver parameters to control in production.

### Example of Driver Design

The driver required for the example in the previous section has the following parameter specifications:

$$f_S = 16.3 \text{ Hz}$$
$$Q_{ES} = 0.336$$
$$V_{AS} = 283 \text{ dm}^3$$
$$V_D = 340 \text{ cm}^3$$
$$P_{E(max)} = 22.5 \text{ W}$$

The driver size will probably have to be at least 12 inches to meet the specifications of $V_D$ and $P_{E(max)}$. This is checked by assuming a typical diaphragm radius of 0.12 m for the 12-inch driver, giving

$$S_D = 4.5 \times 10^{-2} \text{ m}^2.$$

For the required displacement volume of 340 cm³, the peak linear displacement must be

$$x_{max} = V_D/S_D = 7.5 \times 10^{-3} \text{ m} = 7.5 \text{ mm (0.3 in)}.$$

The total "throw" required is then 15 mm (0.6 in) which is realizable in a 12-inch driver. By comparison, the same displacement volume requires a throw of 22 mm (0.9 in) for a 10-inch driver, or 9.6 mm (0.38 in) for a 15-inch driver.

Continuing with the 12-inch design,

$$S_D^2 = 2.0 \times 10^{-3}\,\text{m}^4.$$

The required mechanical compliance and mass are then, from (61) and (62),

$$C_{MS} = 9.9 \times 10^{-4}\,\text{m/N},$$
$$M_{MS} = 97\,\text{g}.$$

$M_{MS}$ is the total moving mass including air loads. Assuming that the front air load is equivalent to that for an infinite baffle and that the driver diaphragm occupies one-third of the area of the front of the enclosure, the mass of the voice coil and diaphragm alone is

$$M_{MD} = M_{MS} - (3.14a^3 + 0.65\pi\rho_o a^3) = 87\,\text{g}.$$

The magnetic damping must be, from (64),

$$B^2l^2/R_E = 30\,\text{N}\cdot\text{s/m (MKS mechanical ohms)}.$$

For an "8Ω" rating impedance, $R_E$ is typically about 6.5 Ω. The required $Bl$ product for the driver is then

$$Bl = 14\,\text{T}\cdot\text{m}$$

which must be maintained with good linearity over the voice-coil throw of 15 mm (0.6 in). The voice coil must also be able to dissipate 22.5 W nominal input power [12, eq. (6)] without damage.

Further examples of driver synthesis based on system small-signal requirements are contained in [28]; the method used is based on the same approach taken above but is arranged for automatic processing by time-shared digital computer. (The Thiele basic efficiency [17] used in this reference is based on a $4\pi$ sr free-field load and gives one-half the value of the reference efficiency used here.)

## 11. DESIGN VERIFICATION

The suitability of a prototype driver designed in accordance with the above methods may be checked by measuring the driver parameters as described in [12].[1] For an air-suspension driver, it is not necessary that $f_S$, $Q_{ES}$, and $V_{AS}$ have exactly the specified values. What is important is that the quantities $f_S^2 V_{AS}$ and $f_S/Q_{ES}$, which together indicate the effective moving mass and electromagnetic coupling, should check with the same combinations of the specified parameters. Then, if $V_{AS}$ is large enough to give a satisfactory value of $\alpha$ for the system, the driver design is satisfactory.

Similarly, the completed system may be checked by measuring its parameters as described in section 6 and comparing these to the initial specifications.[1] The actual system performance may also be verified by measurement in an anechoic environment or by an indirect method [24].

## 12. CONCLUSION

The quantitative relationships presented in this paper make possible the low-frequency design of closed-box systems by direct synthesis from specifications and clearly show whether it is physically possible to realize a desired set of specifications. They should be useful to loudspeaker system designers who wish to obtain the best possible combination of small-signal and large-signal performance within the constraints imposed by a particular design problem.

These relationships should also be useful to driver manufacturers, because they indicate the range of basic driver parameters needed for modern closed-box system design and the extent to which costly magnetic material must be allocated to satisfy both the small-signal and large-signal requirements of the system.

Because the low-frequency performance of a completed system depends on a small number of easily-measured system parameters, it is always possible to specify—and verify—the low-frequency small-signal performance for standard free-field conditions. This information is of much greater value to users of loudspeakers than frequency limits quoted without decibel tolerances and without specification of the acoustic environment.

It is sincerely hoped that the quantitative relationships and physical limitations presented here—and in later papers for other types of direct-radiator systems—will not only be useful to system designers but will also contribute eventually to more uniform, realistic and accurate product specifications.

## 13. ACKNOWLEDGMENTS

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## 14. APPENDIX—SECOND-ORDER FILTER FUNCTIONS

### General Expressions

Tables of filter functions normally give only the details of a low-pass prototype function. The corresponding high-pass or band-pass forms are obtained by suitable transformations. The general form of a prototype low-pass second-order filter function, $G_L(s)$, normalized to unity in the passband, is

$$G_L(s) = \frac{1}{s^2T_0^2 + a_1 sT_0 + 1}, \qquad (68)$$

where $T_0$ is the nominal filter time constant, and the coefficient $a_1$ determines the actual filter characteristic. The corresponding high-pass filter function, $G_H(s)$, which

[1] A recent paper by Benson contains an improved method of $Q$ measurement which compensates for errors introduced by large voice-coil inductance [32, Appendix 2]. The compensation is achieved by replacing $f_C$ in eq. (45) of Part I of this paper— and $f_S$ in [12, eq. (17)]—with the expression $\sqrt{f_1 f_2}$. The measured values of $f_C$ and $f_S$ are unchanged, and no other equations are affected.

preserves the same nominal time constant, is obtained by the transformation

$$G_H(sT_0) = G_L(1/sT_0). \quad (69)$$

This gives the general high-pass expression

$$G_H(s) = \frac{s^2T_0^2}{s^2T_0^2 + a_1sT_0 + 1}. \quad (70)$$

Equations (68) and (70) have exactly the same form as (20) and (19) for the displacement and response functions of the closed-box system. The two sets of equations are equivalent for

$$T_0 = T_C \text{ and } a_1 = 1/Q_{TC}. \quad (71)$$

Study of the steady-state magnitude-*vs*-frequency behavior of filter functions for sinusoidal excitation is facilitated by using the magnitude-squared forms

$$|G_L(j\omega)|^2 = \frac{1}{\omega^4T_0^4 + A_1\omega^2T_0^2 + 1} \quad (72)$$

and

$$|G_H(j\omega)|^2 = \frac{\omega^4T_0^4}{\omega^4T_0^4 + A_1\omega^2T_0^2 + 1}, \quad (73)$$

where

$$A_1 = a_1^2 - 2. \quad (74)$$

### Cutoff Frequency

The half-power frequency $\omega_3 = 2\pi f_3$ of the high-pass function is obtained by setting (73) equal to ½ and solving for $\omega$. Using (71) and (74), the normalized half-power frequency of the closed-box system is given by

$$f_3/f_C = \left[\frac{(1/Q_{TC}^2 - 2) + \sqrt{(1/Q_{TC}^2 - 2)^2 + 4}}{2}\right]^{1/2}. \quad (75)$$

### Frequencies of Maximum Amplitude

The frequency of maximum amplitude of either frequency response or diaphragm displacement is found by taking the derivative of (72) or (73) with respect to frequency and setting this equal to zero. This yields for the normalized frequency of maximum response

$$f_{G\max}/f_C = \frac{1}{[1 - 1/(2Q_{TC}^2)]^{1/2}} \quad (76)$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{G\max}/f_C$ is infinite.

The normalized frequency of maximum diaphragm displacement is

$$f_{X\max}/f_C = [1 - 1/(2Q_{TC}^2)]^{1/2} \quad (77)$$

for $Q_{TC} > 1/\sqrt{2}$. For $Q_{TC} \leq 1/\sqrt{2}$, $f_{X\max}/f_C$ is zero.

### Amplitude Maxima

Substituting the above values of frequency into the expressions for $|G(j\omega)|^2$ and $|X(j\omega)|^2$ corresponding to (72) and (73), the amplitude maxima are found to be

$$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = \left[\frac{Q_{TC}^4}{Q_{TC}^2 - 0.25}\right]^{1/2} \quad (78)$$

for $Q_{TC} > 1/\sqrt{2}$, and unity otherwise.

### Types of Responses

The range of system alignments which may be obtained by varying $Q_{TC}$ are thoroughly described in [13]. Particular alignments of interest, with brief characteristics, are:

Butterworth maximally-flat-amplitude response (*B*2) [13], [29]

$Q_{TC} = 1/\sqrt{2} = 0.707$, $f_3/f_C = 1.000$

Bessel maximally-flat-delay response (*BL*2) [13], [29], [30]

$Q_{TC} = 1/\sqrt{3} = 0.577$, $f_3/f_C = 1.272$

"Critically-damped" response [13]

$Q_{TC} = 0.500$, $f_3/f_C = 1.554$

Chebyshev equal-ripple response (*C*2) [13], [31]

$Q_{TC} > 1/\sqrt{2}$, other properties given by (75)-(78). A very popular alignment of this type is

$Q_{TC} = 1.000$, $f_3/f_C = 0.786$,
$|G(j\omega)|_{\max} = |X(j\omega)|_{\max} = 1.155$ or 1.25 dB.

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**Note:** Dr. Small's biography appeared in the December 1972 issue of the Journal.